Introduction

The development of the incandescent light bulb by Thomas Edison initiated the electrification of modern society and revolutionized the world. This module is intended to introduce the history, fabrication, materials and physics of light bulbs to high school students. Concepts such as Ohm's law, electrical power, energy conservation, electrical resistance and resistivity and blackbody radiation can be enhanced using this unit. Parts of this unit can be tailored for middle school use.

Basic Physics

When a light bulb is turned on using a switch, a constant (ac) voltage V of 120 volts is applied across the filament. Since the filament has a high resistance, because of its fine diameter and long length (see equation 4 below), a small amount of current flows through the filament according to Ohm's law

$$I = \frac{V}{R}, \quad (1)$$

where I is the current through the filament, V is the voltage across the filament and R is the resistance of the filament.

The filament then becomes hot, since the amount of power P produced in the filament is

$$P = \frac{V^2}{R}. \quad (2)$$

It is important to note that since the voltage is a constant in our electrical distribution system, the form of the equation for the power dissipated in a resistor should be that shown in equation 2, and should not be $P=I^2R$. Since V is a constant, equation 2 relates one variable - R - to another variable - P. If the equation $P=I^2R$ was used, then P, I, and R would all be variables.

All objects emit radiation, which was first correctly described by the German physicist Max Planck. The total power emitted per unit surface area (S) of a hot object at a temperature T (in degrees Kelvin) is given by the Stefan-Boltzmann law:

$$\frac{P}{S} = \epsilon\sigma T^4, \quad (3)$$

where the constant □, called the Stefan-Boltzmann constant, has the value 5.67 x 10^{-12} W/(cm^2-K^4). The emissivity of the material □ is a material dependent quantity that can be looked up in a handbook. For tungsten, the value of □ is about 0.3. Note that the symbol □ is also used to represent the electrical conductivity of a material, so that care must be taken in its use so that confusion does not result.

Now for a long cylindrical filament of radius r and length L, the cross sectional area A is □r^2 and the surface area S is 2□rL (ignoring end effects). Recall that the electrical resistance R of material is given by

$$R = \frac{\square L}{A}\ , \quad (4)$$

where □ is the electrical resistivity of the material. For tungsten at room temperature, the value of □ is 5.7 x 10^{-6} ohm-cm.

The final equation that will be used in this unit is the equivalent resistance of 2 resistors in parallel or series. For 2 resistors of resistance R_1 and R_2, the equivalent resistance of their parallel combination R_P is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R_P = \frac{(R_1 \text{ x } R_2)}{(R_1 + R_2)}\ . \quad (5)$$

For 2 resistors in series, the equivalent resistance of the series combination R_S is

$$R_S = R_1 + R_2\ . \quad (6)$$

Equations 1- 6 form the basis for the all of the calculations performed in this unit.

Advanced Topic

Combining equations 2 and 4, we find that the power dissipated in the filament can be written as

$$P=\frac{V^2\pi r^2}{\rho L}. \quad (7)$$

Since the voltage in our houses is fixed, it is apparent from equation 2 that for higher wattage bulbs, the electrical resistance of the filament must decrease as the wattage increases. From equation 4, R can be decreased by increasing A (i.e. r^2) or by decreasing L. Therefore, for higher wattage bulbs, it is necessary to either increase r or decrease L. This may also be seen from equation 7. Since the power will increase as the square of r and only as 1/L, it is simpler to just increase r. Also, it is preferable to increase r since this increases the surface area of the filament. Note that decreasing L decreases the surface area of the filament. This is evident in table 1 below.

From the Stefan-Boltzmann law, the power emitted by the filament can also be written as

$$P=\varepsilon\sigma T^4(2\pi rL). \quad (8)$$

From this equation, we can see that the bulb wattage (brightness), filament temperature (lifetime), filament radius and length are all interdependent. As noted above, it is desirable to maximize the surface area of the filament by increasing r. As can be seen from equation 8, if the power is increased, it is desirable to increase r and L to minimize the increase in T. This is because higher temperatures reduce the lifetime of the filament (although increasing its light output).

From energy conservation considerations, the power dissipated in the filament (power input) must equal the power emitted by the filament (power output). (This neglects other energy loss mechanisms). Therefore equations 7 and 8 can be equated. After some algebra we find that

$$T^4=\frac{V^2 r}{2\rho\varepsilon\sigma L^2} \quad (9)$$

Equation 9 demonstrates the interdependence of the filament temperature and the filament radius and length.