Investigation #1: The Power of (the number) ONE

[Note: the Power of One is a commonly used phrase in advertising and politics, so I thought it would be easy for students to remember. Please do not confuse this title with anything to do with taking a number and raising it to the first power.]

The power of the number one is vastly unappreciated. Its use forms the basis for using and transforming units of measurement – a vitally important concept in science.

Why are units important?

Ask your students what are some reasons for using units.

There are two reasons.

First, the value of a parameter without its corresponding unit is meaningless. Consider the following statements without units:

1. The water temperature is 32.
2. It's about 20 from here.
3. I'll call you back in about 2.

These statements are either unclear or require you to assume a unit of measurement.

They have meaning only when units are included.

1. The water temperature is 32 degrees Celsius.
2. It's about 20 kilometers from here.
3. I'll call you back in about 2 minutes.

If units are not included, the reader must infer the units that you are using. And their inference may be wrong - with serious consequences. In 1999, a confusion between metric and English units caused the loss of the $125 million Mars Climate Orbiter spacecraft.

Consider also this news story dated 11/2/02 from the New York Times News Service::

"Dublin, Ireland – The country will convert all its directional road signs and speed limits to the metric system by the spring of 2004 in compliance with European Union rules, Transport Minister Seamus Brennan said. Speed limits are currently stated in miles per hour, but signs showing distances are notorious for confusing travelers by varying in whether they display distances in miles or kilometers and by not indicating which units are used."

In a 12/20/99 letter to the editor of Design News magazine, Kevin Acheson, Chief Engineer of The Gear Works put it this way, "A number without units is meaningless. In high school I had a Physics teacher who constantly was harping on units. The only way you could pass her class was to show the units with the formulas, as well as with the final answer. At the time it seemed silly, but I did what she required to I could pass. Few things in my education have served me better than the lesson she drove how about always using units. In the 18+ years I have been doing engineering, I have seen more and more young engineers who haven't learned how easy it is to think you have a correct answer only to forget to apply some unit conversion. If anyone thinks that universally switching to the metric system will eliminate stupid errors, they are sadly mistaken. Is a meter the same as a kilometer? Of course not! Without units, no one knows what a number really means.

The second reason is that units provide us with an independent check of the process by which we calculate the answer to our problem; this is called "dimensional analysis." If the units (and magnitude) of the final answer do not make sense, then we know the answer is incorrect. We can also determine the functional relationship between different parameters using dimensional analysis.

Before exploring this aspect of units, let's come back to the power of one.
We know that 1 minute = 60 s, or

$$1 \text{ min} = 60 \text{ s.} \qquad (1)$$

Dividing both sides of equation (1) by 60 s yields an expression (actually an identity) for 1:

$$\frac{1 \text{ min}}{60\text{s}} = 1 \qquad (2)$$

Dividing both sides of the equation (1) by 1 minute yields a complementary expression for 1:

$$\frac{60 \text{ s}}{1 \text{ min}} = 1 \qquad (3)$$

Multiplying or dividing by one does not change the value of a number. So we can always multiply (or divide) by one and not affect the value of the number. If we have a value for some time in seconds and we want to convert it to minutes, we multiply the value by the appropriate expression for one. The procedure is:

A. if you want to convert a unit in the numerator, multiply by the expression for one that has that unit in the denominator.
B. if you want to convert a unit in the denominator, multiply by the expression for one that has that unit in the numerator.

Here's one other major piece of advice to assist in keeping track of units:
ALWAYS USE HORIZONTAL LINES TO SEPARATE FRACTIONS.
(NEVER USE DIAGONAL SLASHES.)

For example:

Convert 120 s to its value in minutes.

$$120 \text{ s} = \frac{120 \text{ s}}{1}$$

We want to convert the unit of s, which is in the numerator, so rule A tells us to use the expression for 1 that has the unit of s in the denominator:

$$\frac{1 \text{ min}}{60 \text{ s}} = 1$$

So:

$$120 \text{ s} = \frac{120 \text{ s} \times 1 \text{ min}}{60 \text{ s}}; \qquad \text{since } 1 \text{ min} \times 1 \text{ s} = 1 \text{ s} \times 1 \text{ min, then}$$

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$$= \frac{120 \text{ min} \times 1 \text{ s}}{60 \quad\quad \text{s}} . \quad \text{Since } \frac{1 \text{ s}}{1 \text{ s}} = 1, \text{ then}$$

120 s = 2 min.