Investigation #6: Driving Safety Solutions

1. How far can a car travelling at 60 mi/h go in:

(a) 0.1 s?

$$\frac{60\text{ mi}}{\text{h}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{5280\text{ ft}}{1\text{ mi}} = \frac{88\text{ ft}}{\text{s}}$$

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 0.1\text{ s} = 8.8\text{ ft}$$

(b) 0.5 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 0.5\text{ s} = 44\text{ ft}$$

(c) 1 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 1.0\text{ s} = 88\text{ ft}$$

(d) 2 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 2.0\text{ s} = 176\text{ ft}$$

(e) 5 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 5.0\text{ s} = 440\text{ ft}$$

2. Travelling at 60 mi/h,

(a) Estimate how far you will travel when you turn around to talk to a friend in the back seat. What are the safety implications of this?

Talking briefly to a friend in the back seat takes about 2 s. In 2 s, you will travel 176 feet (see 1 (d)). This is a long distance so that driving conditions might be drastically different compared to when you last looked at the road, possibly leading to a crash. It is best not to take your eyes off of the road

(b) Estimate how far you will travel when you search for a CD in the glove compartment. What are the safety implications of this?

Reaching and finding a CD in the glove compartment takes about 4 s. In this time, you will travel the following distance:

$$x = v \times t = \frac{88 \text{ ft}}{\text{s}} \times 4.0 \text{ s} = 352 \text{ ft}$$

This is a long distance so that driving conditions might be drastically different compared to when you last looked at the road, possibly leading to a crash. It is best not to take your eyes off of the road.

(c) Estimate how far you will travel when you turn to the side to see if the space next to you is clear for passing. What are the safety implications of this?

Looking to the side takes about 1 s. In 1 s, you will travel 88 feet (see 1 (c)). This is a long distance so that driving conditions might be drastically different compared to when you last looked at the road, possibly leading to a crash. It is best to minimize the time you take your eyes off of the road and it is prudent to leave much more than 88 feet between you and the car in front of you.

(d) The low beams of your headlights will allow you to see about 160 feet in front of you at night. How long does it take your car to travel this distance? What are the safety implications of this?

$$t = \frac{x}{v} = \frac{160 \text{ ft} \ \text{s}}{88 \text{ ft}} = 1.8 \text{ s}$$

You will travel the distance illuminated by your headlights in only 1.8 s. If an object is seen in your headlights, you have at most 1.8 s to avoid it if you stay travelling at 60 mph.

(e) Most drivers need about 1.5 s to react to a new situation. How far will your car travel in this time interval? What are the safety implications of this?

$$x = v \times t = \frac{88\ \text{ft}}{\text{s}} \times 1.5\ \text{s} = 132\ \text{ft}$$

Your car will travel 132 feet in an emergency situation before you even have a chance to hit the brakes!

> Teacher's Note: There is an interesting discussion of reaction times in the book Traffic Safety and the Driver. If you are anticipating an event, reaction times can be as short as 0.15 s. While driving, your reaction time is divided into a perception reaction time ("I need to brake") and a movement reaction time (movement of your foot). For drivers focusing on the car ahead of them, an average reaction time is 1.6 s. For drivers encountering an unexpected obstacle around a blind curve, an average reaction time is closer to 2.5 s.

(f) After the brakes are applied to a car traveling at 60 mi/h, the car needs about 227 feet to stop. Considering reaction time (about 1.5 s) and braking distance, how long a distance will it take to stop if you see a problem up ahead at night? Why does this show the dangers of driving at night?

The car will travel 132 ft while you are reacting to the situation (see 2e above) and then it will take 227 feet to stop. So to stop will require a total stopping distance of 132 ft + 227 ft = 359 ft. Since your headlights illuminate only the road 160 feet in front of you, at 60 mph, you will be unable to stop in time if an obstruction suddenly appears on the road in front of you at night.

Activity Suggestion: Have your students stand near a stoplight. They can determine a typical reaction time by measuring the time it takes for a car to begin moving after a stoplight turns from red to green.

3. How far will a drunk driver's car travel before it stops if it was traveling at 60 mi/hr? What are the safety implications of this? (Recall that the reaction time of a drunk driver is doubled compared to that of a sober driver – so the reaction time of a drunk driver is about 3.0 s)

The reaction time is 3.0 s so the distance the car travels at 60 mph in 3.0 s is:

$$x = v \times t = \frac{88\ \text{ft}}{\text{s}} \times 3.0\ \text{s} = 264\ \text{ft}$$

The braking distance is, as in 2 above, 227 ft.

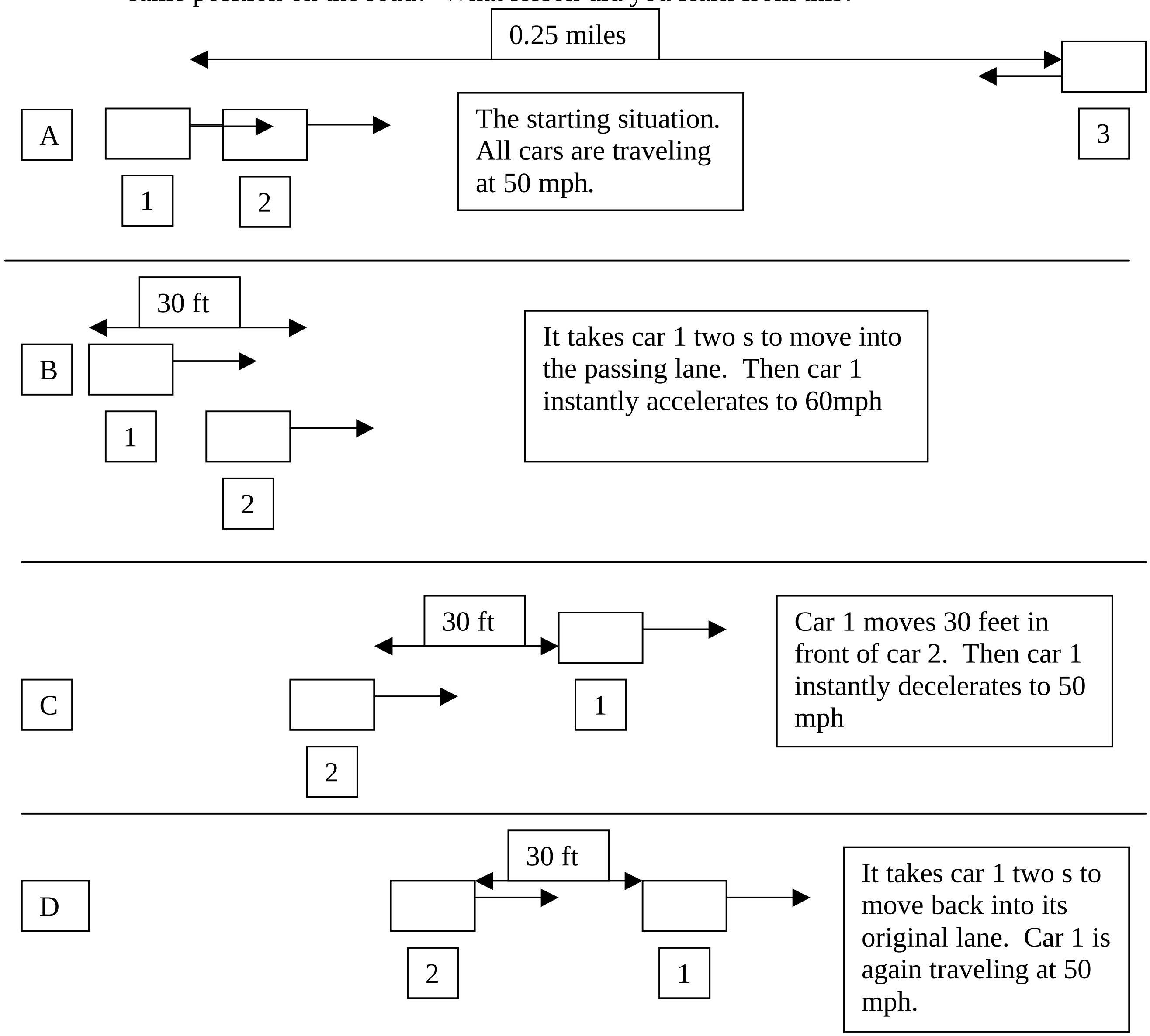
So the total braking distance is 264 ft + 227 ft = 491 ft.

The stopping distance of a drunk driver is 491 ft when traveling at 60 mph. This is (491ft - 359 ft) = 132 ft more than the stopping distance for a sober driver. A drunk driver reacts more slowly than a sober driver, so their car will travel a longer distance before it stops. If there is a problem in the road less than the stopping distance of 491 feet, the drunk driver will hit it. Since the reaction time of a driver on a cell phone is comparable to that of a drunk driver, the above analysis also holds for them.

4. On a 2-lane road (one lane in each direction), you (car 1) decide to pass a car (car 2) in front of you that is traveling at 50 mi/hr. You see another car (car 3) coming towards you from the other direction that is traveling at 50 mi/hr. When you are just behind car 2, you instantly accelerate to 60 mi/hr and move into the other lane. You pass car 2 then move back into your original lane, and instantly decelerate back to 50 mph. Assume that it takes 2 s to travel into the passing lane and 2 s to move back into your original lane. Also assume that you will pass back into your lane when you are two complete car lengths in front of the car you are passing. Each car is 5 m long. The oncoming car is 0.25 miles away from car 1.

a. Will car 1 successfully pass car 2 without hitting the oncoming car 3?

a. If so, how many seconds later would your car and the oncoming car be at the same position on the road? What lesson did you learn from this?



a. Car 1 must travel 60 feet more than car 2 to be able to pass it. Car 1 is travelling 10 mph (15 ft/s) faster than car 2 while it is in the passing lane. So car 1 will take the following amount of time to pass car 2.

$$t = \frac{x}{v} = \frac{60 \text{ ft s}}{15 \text{ ft}} = 4 \text{ s}$$

So car 1 takes 2 s to move into the passing lane (at 50 mph), 4 s to pass car 1 (at 60 mph), and then 2 more s to move back into its original lane (at 50 mph), the total time is 8 s.

In 8 s, car 1 will travels for 2 s at 50 mph, then for 4 s at 60 mph, then 2 s for 50 mph, so the total distance traveled by car 1 is:

$$x = \frac{73.3 \text{ ft}}{\text{s}} \times 2 \text{ s} + \frac{88 \text{ ft}}{\text{s}} \times 4 \text{ s} + \frac{73.3 \text{ ft}}{\text{s}} \times 2 \text{ s} = 645 \text{ ft}$$

In 8 s, car 3 will travel the following distance (towards car 1)

$$x = \frac{73.3 \text{ ft}}{\text{s}} \times 8 \text{ s} = 586 \text{ ft}$$

So at the end of 8 s car 1 and car 2 are 645 ft + 586 ft = 1231 ft closer. The cars started 0.25 miles apart, which is 1320 ft (recall that 1 mile = 5280 ft). When car 1 is back in its original lane after passing car 2, car 1 will be only 89 feet from car 3. Since their relative speed of approach is 50 mph + 50 mph = 100 mph = 147 ft/s, the safety margin for passing car 2 was less than 1 s.

b. Even though car 1 started to pass car 2 when car 3 was 0.25 miles away, car 1 and car 3 missed colliding at 50 mph by less than 1 s. So you must allow much more distance and time for passing than is obvious.

> Excerpt from Traffic Safety and the Driver (p. 118): "It is found that while drivers make reliable estimates of the distance to the oncoming car, they are insensitive to its speed... The inability of drivers to estimate oncoming speed leads them to decline safe passing opportunities when the oncoming car is travelling slower than expected, and to initiate unsafe passing maneuvers when the oncoming car is travelling faster than expected."

Note: As an extension, students could also plot the position vs. time and the velocity (or speed) vs. time for each of the three cars.

5. Suppose you a traffic engineer working for the California Department of Transportation. Your job is to set the timing of traffic lights. You are also to only use metric units. How long should traffic lights be yellow for the following speeds: 45 km/hr, 65 km/hr, 85 km/hr, and 105 km/hr? Use the results of problem 2 in the simulation tool development section.

First translate the metric speeds to English unit speeds.

a. $\frac{45 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 28 \text{ mph}$

b. $\frac{65 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 41 \text{ mph}$

c. $\frac{85 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 53 \text{ mph}$

d. $\frac{105 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 66 \text{ mph}$

Using the table of total stopping times in problem 2 in the simulation tool development section to estimate the total stopping times at the 4 speeds.

Speed (km/h)	Total stopping time (s) from problem 2 in simulation tool development section	California traffic manual suggested times for yellow lights
45	3.9	3.1
65	5.0	3.9
85	6.0	4.9
105	7.2	5.8

The yellow light times suggested by the California traffic manual seem to be about 1 second less than would be expected. Let's explore this further in the activities below.

Suggested student activity: Have your students measure the duration of yellow lights. For a certain speed limit, are they always set the same? How are they set at different speed limits?

e. What is the distance traveled by car during California department of transportation yellow light time if it stays at its initial speed.

At 28 mph:

$\frac{28\text{mi}}{\text{hr}} \times \frac{88\text{ft}}{\text{s}} \frac{\text{hr}}{60\text{ mi}} \times 3.1\text{ s} = \frac{41.1\text{ ft}}{\text{s}} \times 3.1\text{ s} = 127\text{ ft}$

At 41 mph:

$\frac{41\text{mi}}{\text{hr}} \times \frac{88\text{ft}}{\text{s}} \frac{\text{hr}}{60\text{ mi}} \times 3.9\text{ s} = \frac{60.1\text{ ft}}{\text{s}} \times 3.9\text{ s} = 234\text{ ft}$

At 53 mph:

$\frac{53\text{mi}}{\text{hr}} \times \frac{88\text{ft}}{\text{s}} \frac{\text{hr}}{60\text{ mi}} \times 4.9\text{ s} = \frac{77.7\text{ ft}}{\text{s}} \times 4.9\text{ s} = 381\text{ ft}$

At 66 mph:

$\frac{66\text{mi}}{\text{hr}} \times \frac{88\text{ft}}{\text{s}} \frac{\text{hr}}{60\text{ mi}} \times 5.8\text{ s} = \frac{96.8\text{ ft}}{\text{s}} \times 5.8\text{ s} = 561\text{ ft}$

f. What is the stopping distance at the above 4 speeds:

Recall that the otal stopping distance = reaction distance + braking distance

$$= v\,(1.5\text{ s}) + \frac{v^2}{2a}$$

At 28mph:

Total stopping distance = $\frac{41.1\text{ ft}}{\text{s}} \times 1.5\text{ s} + \frac{(41.1\text{ ft})^2}{\text{s}^2} \frac{\text{s}^2}{(2)(17\text{ ft})} = 111\text{ ft}$

At 41mph:

Total stopping distance = $\frac{60.1\text{ ft}}{\text{s}} \times 1.5\text{ s} + \frac{(60.1\text{ ft})^2}{\text{s}^2} \frac{\text{s}^2}{(2)(17\text{ ft})} = 197\text{ ft}$

At 53mph:

Total stopping distance = $\frac{77.7\text{ ft}}{\text{s}} \times 1.5\text{ s} + \frac{(77.7\text{ ft})^2}{\text{s}^2} \frac{\text{s}^2}{(2)(17\text{ ft})} = 294\text{ ft}$

At 66mph:

Total stopping distance = $\frac{96.8\text{ ft}}{\text{s}} \times 1.5\text{ s} + \frac{(96.8\text{ ft})^2}{\text{s}^2} \frac{\text{s}^2}{(2)(17\text{ ft})} = 373\text{ ft}$

g. Calculate the difference between the constant speed distance and the total stopping distance determined in e and f above.

At 28mph:
Constant speed distance - Total stopping distance
=127 ft - 111 ft = 16 ft

At 41mph:
Constant speed distance - Total stopping distance
=235 ft - 197 ft = 38 ft

At 53mph:
Constant speed distance - Total stopping distance
=381 ft - 294 ft = 87 ft

At 66mph:
Constant speed distance - Total stopping distance
=561 ft - 373 ft = 188 ft

h. Determine the time it takes to travel the distance determined in g. This is the amount of time you have to decide to brake so that you can stop before the light.

Time to travel constant speed distance - total stopping distance
At 28mph:

$$\frac{16\text{ ft s}}{41.1\text{ ft}} = 0.4\text{ s}$$

So at 28mph, you have 0.4 s of safety margin to decide to brake.

At 41mph:

$$\frac{38\text{ ft s}}{60.1\text{ ft}} = 0.6\text{ s}$$

So at 41mph, you have 0.6 s of safety margin to decide to brake.

At 53mph:

$$\frac{87\text{ ft s}}{77.7\text{ ft}} = 1.1\text{ s}$$

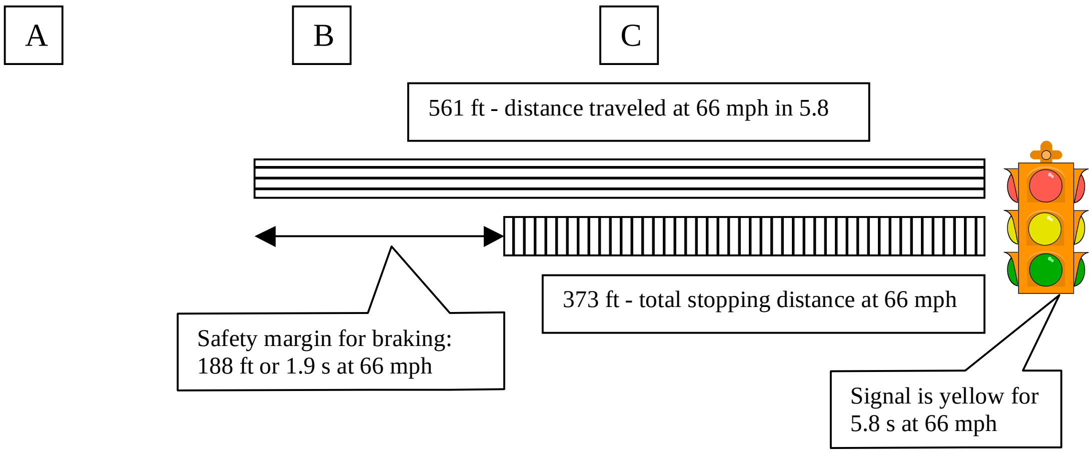
So at 53mph, you have 1.1 s of safety margin to decide to brake.

At 66mph:

$\frac{188 \text{ ft } s}{96.8 \text{ ft}} = 1.9 \text{ s}$

So at 66mph, you have 1.9 s of safety margin to decide to brake.

i. Draw a diagram showing the distance traveled at 66 mph, the total stopping distance at 66 mph, and the safety margin in distance and time you have to decide to brake.



At point A, you cannot make it through the light traveling at 66 mph so you should brake and stop. You should know from experience that your car can easily and safely brake at this distance from the light.

At point B, you can either drive at the same speed or brake. You will pass the light before it turns red if you continue traveling at the speed limit. You also have enough time to safely brake if you decide to stop before the light. So at this critical distance from the light, either decision will be a safe one.

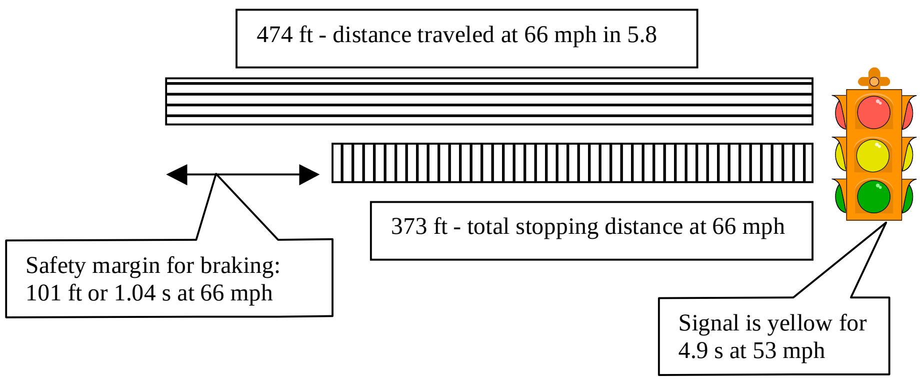
At point C, you do not have enough distance to brake before the light. You should know from experience that your car cannot safely brake at this distance from the light. You should continue traveling at the same speed - you will easily pass the light before it turns red.

> From the San Diego Union Tribune 10/6/01, page B1.
> Headline: "Davis signs bill on red-light cameras; yellow to be timed."
>
> "Gov. Gray Davis signed a bill yesterday requiring that traffic lights with cameras show the yellow caution light for a reasonable amount of time before taking photos of red-light violators.
>
> The amount of time for triggering the automatic cameras would be determined by the California Department of Transportation,
>
> There have been allegations that some traffic lights in San Diego and other areas were set to switch too swiftly from green to red.
>
> SB 667 by Sen. Steve Peace requires that traffic lights with cameras comply with the Caltrans traffic manual for minimum yellow-light intervals.
>
> The minimum depends on the speed limit. For example, at 25 mph the yellow light must be on for at least 3 seconds. At 45 mph, the interval must be at least 4.3 seconds."

j. Redo the diagram of part i assuming that you are speeding: you are traveling at 66 mph in a zone where the speed limit is 53 mph. At 53 mph, the signal is yellow for 4.9 s.

$$\text{Distance} = 66\text{mph} \times \frac{88\text{ft/s}}{60\text{ mph}} \times 4.9\text{ s} = 474\text{ ft}; \quad 66\text{ mph} \times \frac{88\text{ft/s}}{60\text{ mph}} = 96.8\text{ ft/s}$$

Safety margin time = (474ft - 373ft)/(96.8ft/s) = 1.04 s

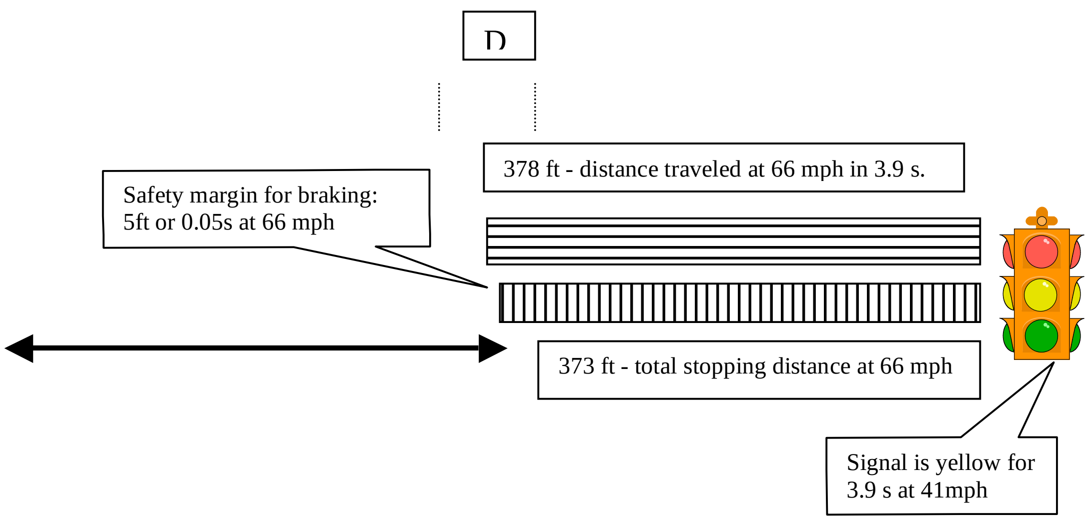


k. Redo the diagram of part i assuming that you are speeding: you are traveling at 66mph in a zone where the speed limit is 41 mph. At 41 mph, the signal is yellow for 3.9 s. What do these results indicate about the dangers of speeding.

$$\text{Distance} = 66\text{mph} \times \frac{88\text{ft/s}}{60\text{ mph}} \times 3.9\text{ s} = 378\text{ ft}; \quad 66\text{ mph} \times \frac{88\text{ft/s}}{60\text{ mph}} = 96.8\text{ ft/s}$$

Safety margin time = (378ft - 373ft)/(96.8ft/s) = 0.05 s

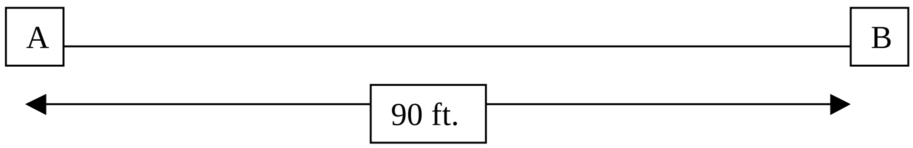
If the car is in the hatched in area shown in the diagram below, it must continue through the light. If it is in the area shown by the arrow, it must stop in order to not run the red light. There is no area B as shown in part i of this problem since the safety margin for making the correct decision is now only 5 ft or 0.05 s when traveling at 66 mph. Near the transition region D, the driver must make an immediate correct decision as to whether or not they should hit the brakes or maintain their speed.



6. A general rule of thumb taught to drivers is to leave one car length of distance between cars for every 10 mph. Assume that a typical car length is 15 feet or 3 meters. Does this make sense? Why or why not? Consider 3 different cases.

a. You are traveling at 60 mph and suddenly the traffic in front of you slows to 50 mph.

The situation is as shown below:



First calculate how long it takes car A to brake from 60 mph to 50 mph.
It takes a driver about 1.5 s to react, so car A travels the following distance:

$$x = vt = \frac{88 \text{ ft}}{\text{s}} \times 1.5 \text{ s} = 132 \text{ ft.}$$

Next determine the time it takes a car to slow down from 60 mph to 50 mph at a constant rate of deceleration of 17 ft/s^2. Previously, we found that 50 mph = 73.3 ft/s.

Since $a = \frac{v_f - v_I}{t}$, then $t = \frac{v_f - v_I}{a} = \frac{(73.3 \text{ ft} - 88 \text{ ft})\ \text{s}^2}{\text{s} \times (-17 \text{ ft})} = 0.86 \text{ s}$

The average speed of car A during this time is $\frac{v_f + v_I}{2} = \frac{81 \text{ ft.}}{\text{s}}$

So car A travels $x = vt = \frac{81 \text{ ft}}{\text{s}} \times 0.86 \text{ s} = 70 \text{ ft}$ as it decelerates to 50 mph.

So the total time it takes car A to slow down to 50 mph = 1.5 s + 0.86 s = 2.36 s.
The total distance it travels is 132 ft + 70 ft = 202 ft.

In this time interval, car B will travel $x = vt = \frac{73.3 \text{ ft}}{\text{s}} \times 2.36 \text{ s} = 182 \text{ ft.}$

When both cars are traveling at 50 mph, car A will have traveled (202 ft – 182 ft) = 20 feet more than car B. Since this distance is less than 90 ft, car A will not come too close to car B – car A will end up 70 feet behind car B.

b. You are traveling at 60 mph and suddenly the traffic in front of you slows to 40 mph.

First calculate how long it takes car A to brake from 60 mph to 40 mph.
It takes a driver about 1.5 s to react, so car A travels the following distance:
$x = vt = \frac{88 \text{ ft}}{\text{s}} \times 1.5 \text{ s} = 132 \text{ ft}.$

Next determine the time it takes a car to slow down from 60 mph to 40 mph at a constant rate of deceleration of 17 ft/s^2. Previously, we found that 40 mph = 49 ft/s.

Since $a = \frac{v_f - v_I}{t}$, then $t = \frac{v_f - v_I}{a} = \frac{(49 \text{ ft} - 88 \text{ ft})\text{ s}^2}{\text{s} \times (-17 \text{ ft})} = 2.3 \text{ s}$

The average speed of car A during this time is $\frac{v_f + v_I}{2} = \frac{68.5 \text{ ft.}}{\text{s}}$

So car A travels $x = vt = \frac{68.5 \text{ ft}}{\text{s}} \times 2.2 \text{ s} = 151 \text{ ft}$ as it decelerates to 40 mph.

So the total time it takes car A to slow down to 50 mph = 1.5 s + 2.3 s = 3.8 s. The total distance it travels is 132 ft + 151 ft = 283 ft.

In this time interval, car B will travel $x = vt = \frac{49 \text{ ft}}{\text{s}} \times 3.8 \text{ s} = 186 \text{ ft}.$

When both cars are traveling at 40 mph, car A will have traveled (283 ft – 186 ft) = 97 feet more than car B. Since this distance is more than 90 ft, car A will hit car B.

c. You are travelling at 60 mph and all of a sudden you become aware that the traffic is stopped in front of you when the traffic is only 6 car lengths in front of you.

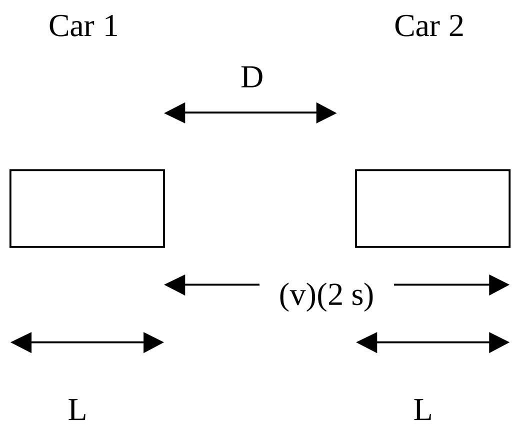
The total stopping distance at 60 mph is 360 ft (see problem 5f), much more than the distance of 6 car lengths of 90 ft. So you will hit the car in front of you. It takes you 1.5 s to react to the situation. During this time, your car travels 132 ft, so you will still be traveling at 60 mph when you hit the car in front of you – which will probably result in serious injury or death.

In conclusion, as long as you keep your relative speed to within about 15 mph of the traffic in front of you, you will be able to react in time.

7. Another way to judge appropriate spacing between cars when driving is to consider what is called "headway." Headway is defined as the elapsed time between the front of the lead vehicle passing a point on the roadway and the front of the following vehicle passing

the same point. Most driving manuals recommend a headway of at least 2 s. How does a headway of 2-s compare to the rule of thumb that you should leave 1 car length between the front of your car and the back of the car in front of you for every 10-mph of speed?
Hint 1: Assume that each car length is 14.7 ft long. Recall that 10mph = 14.7 ft/s.
Hint 2: Assume two cars are moving at the same constant speed, one behind the other. Call the speed v. At time t = 0 the front of the lead vehicle passes a given point on the highway. Two seconds later the front of the second vehicle passes that same point. The total distance between the front of the two vehicles is therefore (v)(2 s).

Assume two cars are moving at the same constant speed, one behind the other. Call the speed v. At time t = 0 the front of the lead vehicle passes a given point on the highway. Two seconds later the front of the second vehicle passes that same point. The total distance between the front of the two vehicles is therefore (v)(2 s). Thus the distance D between the back of the first car and the front of the second is (v) (2 s) – L, where L is the length of the car.



The lengths of cars vary, typically ranging around 14-15 ft. For simplicity, take L = 14.7 ft. This gives us

$$D = (v)(2\ s) - L = (v)(2\ s) - 14.7\text{ft}$$

For v = 10mph, v = 14.7 ft/s
so

$$D = \frac{(14.7\ \text{ft})(2\ \text{s})}{\text{s}} - 14.7\ \text{ft} = 14.7\ \text{ft} = 1\ \text{car length}$$

For v = 60mph, v = 88 ft/s
so

$$D = \frac{(88ft)(2\ s)}{s} - 14.7\ ft = 161.3\ ft$$

$$\frac{161.3\ ft \times 1\ car\ length}{14.7\ ft} = 11\ car\ lengths$$

Thus, the distance between cars varies from 1 car length for every 10mph (at low speeds) to a bit less than 2 car lengths for every 10mph (at high speeds), which is in accord with the rough rule of thumb but is better because it allows for greater spacing at higher speeds.

Not only is a headway of 2 seconds a safer rule of thumb, you can more readily measure headway on the road. Estimating the distance between vehicles is more difficult than noting the time between cars passing the same point on the road.

Excerpt from Traffic Safety and the Driver (p. 314-316): "... drivers who are following other vehicles do so with an average headway of 1.32 seconds; that is, the average headway is considerably shorter than the recommended minimum. ... Why do drivers choose to follow so closely? It seems to me that it becomes largely a driving habit, rather than reasoned conscious behavior. ... Following at a headway of 2.0 seconds instead of 0.5 seconds means you will arrive 1.5 seconds later, assuming that no vehicles cut in front of you. ... Even if a few vehicles do cut into the gap in front ... this adds only about 2 seconds per such incident to the overall trip time. Drivers probably object to other vehicles cutting in front of them not because it delays them a couple of seconds, but because it is interpreted as some sort of personal affront, an assault on manhood or womanhood. If detached rationality cannot dispel such feelings, comfort might be sought in the confident expectation that the offending driver is likely to be experiencing more than the average crash rate of one per 10 years. Let such drivers have their fun - they are paying a high price for it; recapture your two seconds by walking faster to your vehicle."

8. You are driving at night to a friend's house. You make a sharp right hand turn onto another street and suddenly, 75 feet in front of you, you see someone crossing the street. Based on the results of problem 5, what is the maximum speed you could be traveling at and not hit the person.

Based on problem 5, the maximum speed is about 20 mph, since at 20 mph, the total stopping distance is about 70 ft.

How fast should you be driving when making a turn onto a street?

You speed should be such that your stopping distance is less than the distance your lights are illuminating in front of you. When making a turn, that distance may be as little as 25 feet, so you should not make turns at night at speeds higher than about 10 mph.

9. Headlights illuminate the road up to 160 feet in front of you. If you are on a road with stop signs, what is the fastest speed you can drive and still stop safely at night?

The total stopping distance must be less than or equal to 160 feet we will assume it is equal to 160 ft so that your car will come to a complete stop right at the stop sign. If you are initially travelling at a speed v, then the reaction distance is vt and the braking distance is $v^2/(2a)$.

So the total stopping distance (d_{total}) is:

$$d_{total} = vt + \frac{v^2}{2a}.$$

d_{total} = 160 ft, the braking deceleration $a = \frac{17 \text{ ft}}{s^2}$, and the reaction time t = 1.5 s.

The equation that must be solved is a quadratic equation:

$$\frac{v^2}{2a} + vt - d_{total} = 0 \text{ or } \frac{0.029 \, s^2 v^2}{ft} + 1.5s \, v - 160ft = 0$$

Using the quadratic formula

$$v = \frac{-1.5 \text{ s +/- sqr } \{1.5^2 s^2 - 4 * .029 s^2/ft * (-160ft)\}}{2 * 0.029 \, s^2/ft}$$

Only the + sign yields a physically meaningful solution, so:

$$v = \frac{52.8 \text{ ft}}{s} \text{ or 36 mph.}$$

Perhaps this is why the speed limit on country roads, where you may encounter a stop sign without warning, is often 35 mph.
If you were drunk and traveling at this speed, your reaction time would double to 3 s. So your reaction distance would be:

$$\text{Reaction distance} = vt = \frac{52.7 \text{ ft}}{s} \text{ x } 3 \text{ s} = 158.1 \text{ ft.}$$

$$\text{Your braking distance is still } \frac{v^2}{2a} = \frac{52.7^2 \text{ ft}^2 \, s^2}{s^2 \, 2 \text{ x } 17 \text{ ft}} = 81.7 \text{ ft}$$

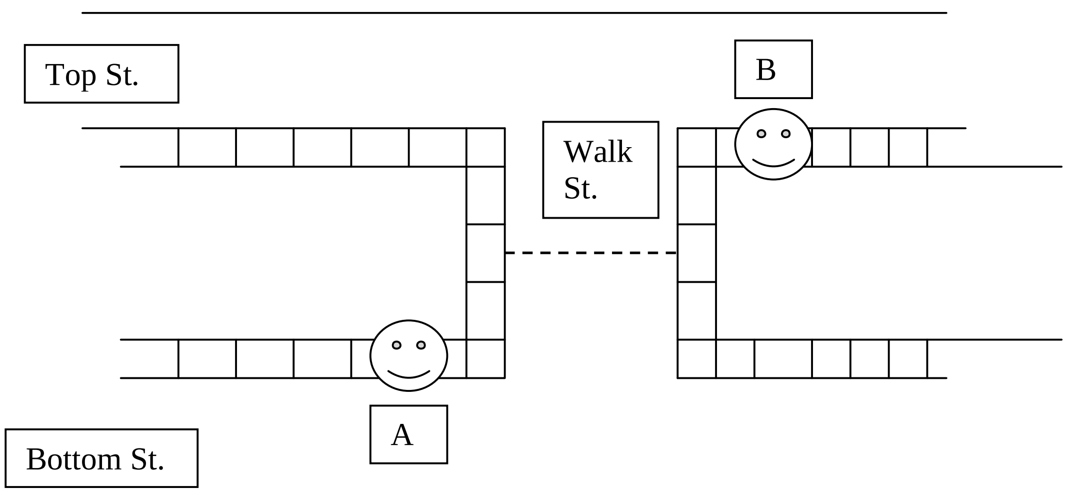
So the total braking distance = 151.8 ft + 81.7 ft = 239.8 ft. This distance is greater than you can see using your lights (160 ft), so you would not be able to stop before the stop sign. You will be traveling through the intersection, possibly causing a crash.

9A. You are traveling on a freeway with a small amount of traffic. After about a hour of driving, you notice that there appear to be no cars traveling at your speed - all cars seem to be either passing you or you are passing them. Why is this?

The reason is that cars traveling at the same speed as you never get closer to you or farther from you - they stay the same distance from you. So you will never see the cars traveling at your speed.

9B. Smiley face wants to walk from the sidewalk at point A to the sidewalk at point B at night. She needs to cross the street. Where is the safest place for Smiley to cross and why?

Smiley should cross at the dashed line since it gives her the most time to react to a car entering Walk St. from either Top St. or Bottom St.



10. A fire engine is traveling at 25 m/s directly towards a bus station on its way to a fire. At its closest approach it passes right next to the station. Starting at 400 m before the station, it sends out a very short blast of sound every 100 m. It stops sending these messages when it is 400 m past the station. Sound travels at 330 m/s. If you are standing at the bus station, determine the time interval between successive blasts of sound. Calculate and compare (using a table and a chart) how the time intervals change when the fire engine is approaching you versus when it is moving away from you.

At time t=0, the first sound is sent out. At this point, the fire engine is 400 meters away, so the sound takes $\Delta t = d/v = 400\ s/330 = 1.2$ s. So the first sound appears at the station at t = 1.2 s.

After the fire engine has traveled 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 4 s. The sound must now travel 300 m, which takes $\Delta t = d/v = 300\ s/330 = 0.9$ s. So this sound appears at the station at t = 4.9 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 8 s. The sound must now travel 200 m, which takes $\Delta t = d/v = 200\ s/330 = 0.6$ s. So this sound appears at the station at t = 8.6 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 12 s. The sound must now travel 100 m, which takes $\Delta t = d/v = 100\ s/330= 0.3$ s. So this sound appears at the station at t = 12.3 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. It is now at the station. So this sound is sent out at t = 16 s. The sound must now travel 0 m, which takes $\Delta t = d/v = 0\ s/330 = 0$ s. So this sound appears at the station at t = 16 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. It is now 100 m past the station. So this sound is sent out at t = 20 s. The sound must now travel 100 m, which takes $\Delta t = d/v = 100\ s/330 = 0.3$s. So this sound appears at the station at t = 20.3 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 24 s. The sound must now travel 200 m, which takes $\Delta t = d/v = 200\ s/330 = 0.6$ s. So this sound appears at the station at t = 24.6 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 28 s. The sound must now travel 300 m, which takes $\Delta t = d/v = 300\ s/330 = 0.9$ s. So this sound appears at the station at t = 28.9 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 32 s. The sound must now travel 400 m, which takes $\Delta t = d/v = 400\ s/330 = 1.2$ s. So this sound appears at the station at t = 33.2 s.

Make a table summarizing the data.

Distance of fire engine from station (m)	Time signal arrived at station (s)	Time between successive signals (s)
-400	1.2	
-300	4.9	3.7
-200	8.6	3.7
-100	12.3	3.7
0	16.0	3.7
100	20.3	4.3
200	24.6	4.3
300	28.9	4.3
400	33.2	4.3

So the time between successive blasts of sounds is less as the fire engine is approaching the station compared with the time between successive blasts of sound as the fire engine travels away from the station. This is the origin of the Doppler effect.

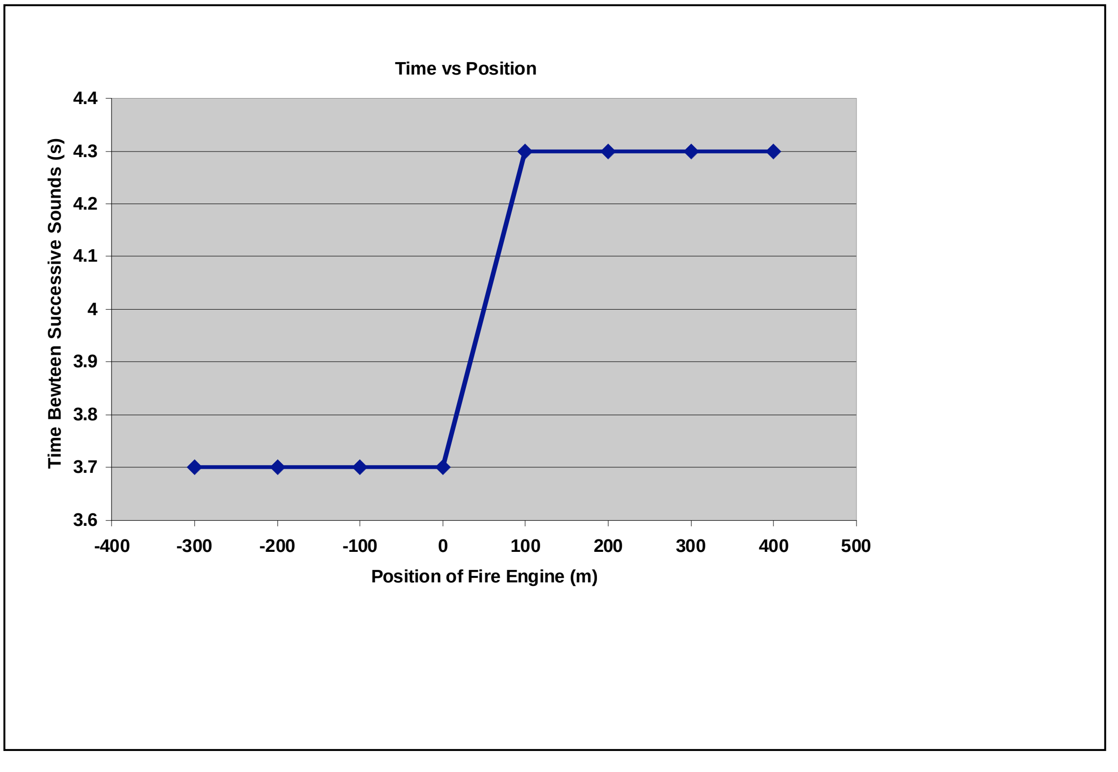
Note that there is a difference between the fire engine approaching the station and receding from the station.

As the fire engine is approaching the station:

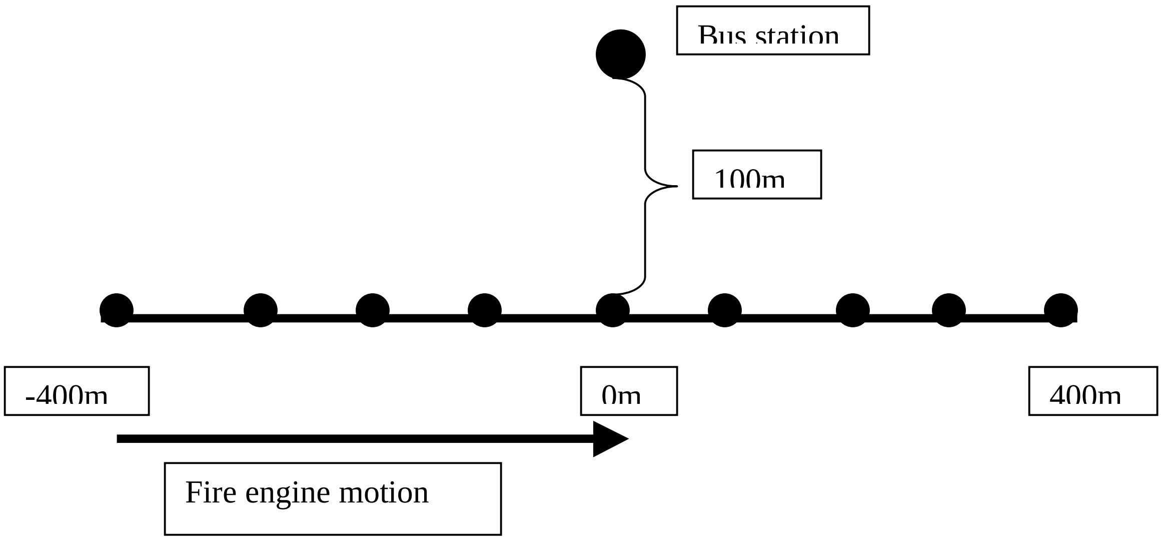
- it sends out a signal
- it travels toward the station
- it sends out the next signal when it is closer to the station

As the fire engine is receding from the station:

- it sends out a signal
- it travels away from the station
- it sends out the next signal when it is farther from the station



10A. A fire engine is traveling at 25 m/s on its way to a fire. At its closest approach it passes 100m from a bus station. Starting at 400 m before the station, it sends out a very short blast of sound every 100 m. It stops sending these messages when it is 400 m past the station. Sound travels at 330 m/s. If you are standing at the bus station, determine the time interval between successive blasts of sound. Calculate and compare (using a table and a chart) how the time intervals change when the fire engine is approaching you versus when it is moving away from you.



At time t=0, the first sound is sent out. At this point, the fire engine is $(400^2 + 100^2)^{0.5}$ m = 412.3 m away, so the sound takes $\Delta t = d/v$ = 412.3s/330 = 1.249 s. So the first sound appears at the station at t = 1.249 s.

After the fire engine has traveled 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 4 s. The sound must now travel $(300^2 + 100^2)^{0.5}$ m = 316.2 m, which takes $\Delta t = d/v$ = 316.2 s/330 = 0.958 s. So this sound appears at the station at t = 4.958 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 8 s. The sound must now travel $(200^2 + 100^2)^{0.5}$ m = 223.6 m, which takes $\Delta t = d/v$ = 223.6 s/330 = 0.678 s. So this sound appears at the station at t = 8.678 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 12 s. The sound must now travel $(100^2 + 100^2)^{0.5}$ m = 141.4 m, which takes $\Delta t = d/v$ = 141.4 s/330= 0.428 s. So this sound appears at the station at t = 12.428 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. It is now at its closest approach to the bus station. So this sound is sent out at t = 16 s. The sound must now travel 100 m, which takes $\Delta t = d/v$ = 100 s/330 = 0.303 s. So this sound appears at the station at t = 16.303 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 20 s. The sound must now travel $(100^2 + 100^2)^{0.5}$ m = 141.4 m, which takes $\Delta t = d/v$ = 141.4 s/330= 0.428 s. So this sound appears at the station at t = 20.428 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 24 s. The sound must now travel $(200^2 + 100^2)^{0.5}$ m = 223.6 m, which takes $\Delta t = d/v$ = 223.6 s/330 = 0.678 s. So this sound appears at the station at t = 24.678 s.

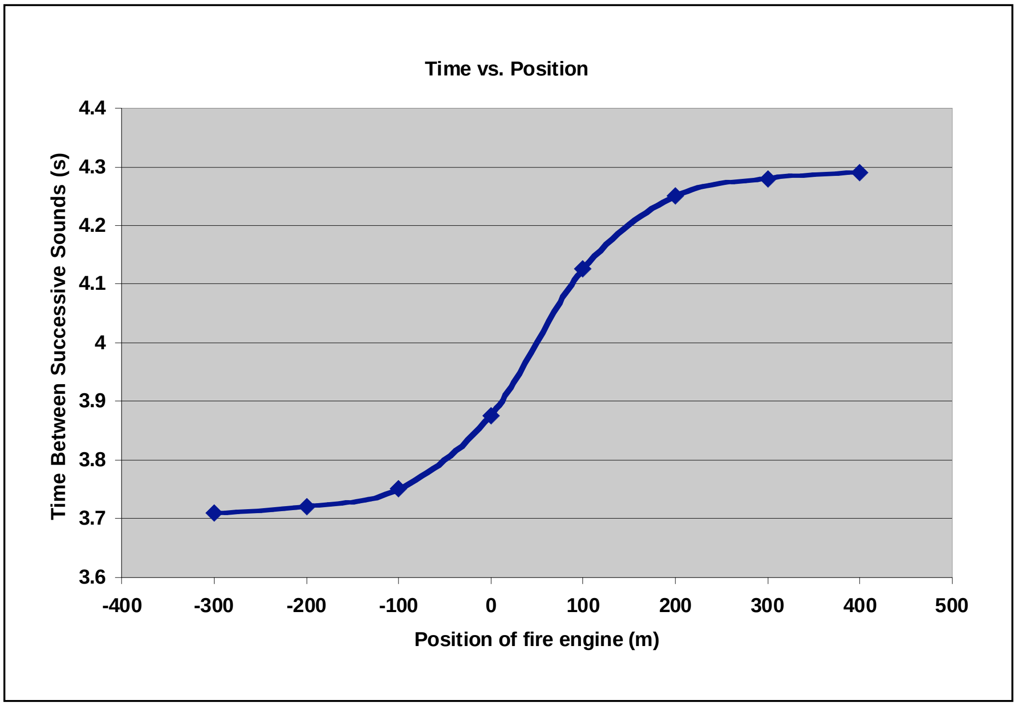
After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 28 s. The sound must now travel $(300^2 + 100^2)^{0.5}$ m = 316.2 m, which takes $\Delta t = d/v$ = 316.2 s/330 = 0.958 s. So this sound appears at the station at t = 28.958 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 32 s. The sound must now travel $(400^2 + 100^2)^{0.5}$ m = 412.3 m, so the sound takes $\Delta t = d/v$ = 412.3s/330 = 1.249 s. So this sound appears at the station at t = 33.249 s.

Make a table summarizing the data.

Distance of fire engine from station (m)	Time signal arrived at station (s)	Time between successive signals (s)
-400	1.249	
-300	4.958	3.709
-200	8.678	3.720
-100	12.428	3.750
0	16.303	3.875
100	20.428	4.125
200	24.678	4.250
300	28.958	4.280
400	33.249	4.291

So the time between successive blasts of sounds slowly increases as the fire engine is approaching the bus station, then increases rapidly as it passes its closest approach to the bus station, then slowly increases again as it travels away from the bus station. This is the origin of the Doppler effect.



11. Jack and Jill each drive their vehicles 10,000 miles per year. Jack's vehicle has a fuel economy of 10 miles per gallon, Jill's 30 miles per gallon.

a. How much fuel does each of them use in a year?

Jack: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{10 \text{ mi}} = \frac{1000 \text{ gallons}}{\text{year}}$

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{30 \text{ mi}} = \frac{333 \text{ gallons}}{\text{year}}$

b. How much fuel does the Jack and Jill household use in a year?

The Jack and Jill household uses 1333 gallons per year.

c. How far do they travel in a year?

The Jack and Jill household travels a total of 20,000 miles.

d. What is their average household fuel economy? Is it the average of Jack's fuel economy and Jill's fuel economy?

The Jack and Jill household average fuel economy is given by the total miles driven divided by the total fuel used, so

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1333 \text{ gallons}} = \frac{15 \text{ miles}}{\text{gallon}}$.

Note that this is NOT the average of Jack's fuel economy and Jill's fuel economy, which would be 20 miles per gallon.

e. What would their average household fuel economy be if Jill's vehicle got 100 miles per gallon?

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{100 \text{ mi}} = \frac{100 \text{ gallons}}{\text{year}}$

So the total household would use 1100 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1100 \text{ gallons}} = \frac{18.2 \text{ miles}}{\text{gallon}}$.

f. What would their average household fuel economy be if Jill's vehicle got 1000 miles per gallon?

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{1000 \text{ mi}} = \frac{10 \text{ gallons}}{\text{year}}$

So the total household would use 1010 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1010 \text{ gallons}} = \frac{19.8 \text{ miles}}{\text{gallon}}$.

g. What would their average household fuel economy be if Jill's vehicle got 10,000 miles per gallon?

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{10{,}000 \text{ mi}} = \frac{1 \text{ gallon}}{\text{year}}$

So the total household would use 1001 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1001 \text{ gallons}} = \frac{19.98 \text{ miles}}{\text{gallon}}$.

h. What would their average household fuel economy be if Jack's vehicle got 30 miles per gallon, the same as Jill's original vehicle?

Jack: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{30 \text{ mi}} = \frac{333 \text{ gallon}}{\text{year}}$

So the total household would use 666 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{666 \text{ gallons}} = \frac{30 \text{ miles}}{\text{gallon}}$.

i. If you were in charge of making policy to reduce fuel consumption, what would you do?

It is far better to try to improve the fuel economy of the worst vehicles than the best vehicles. Therefore, you might try to make a minimum fuel economy standard or, as Congress has done, mandate a required average fuel economy for all vehicles sold by a car manufacturer.

12. Consider the following distribution of dots on the line below. Let's call the dots "galaxies" and let's call the line "the universe." Suppose that adjacent galaxies are all located a distance of L apart from each other in the universe. At a time T later, the universe has expanded a factor of two so that now all of the adjacent galaxies are a distance of 2L apart.

a. Suppose you are living in galaxy A. How fast does it appear that galaxies B, C, and D are receding from you?

From the perspective of galaxy A, galaxy B traveled a distance of L in time T so its speed of recession is L/T. From the perspective of galaxy A, galaxy C traveled a distance of 2L in time T so its speed of recession is 2L/T. From the perspective of galaxy A, galaxy D traveled a distance of 3L in time T so its speed of recession is 3L/T.

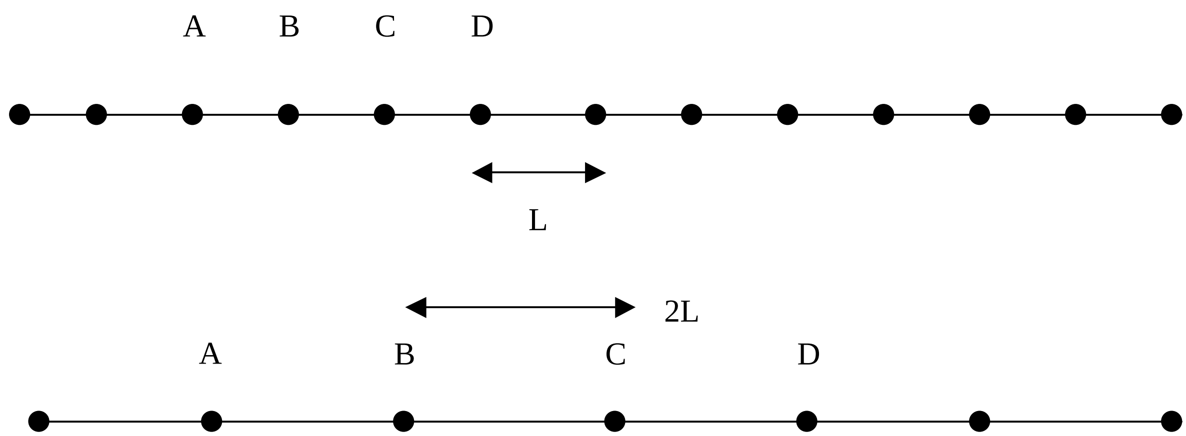
b. Is there a correlation between the distance the galaxy is located from you and the speed with which it is receding from you. What is that relationship?

The further away the galaxy is from you, the faster it appears to be moving. The relationship (known as the Hubble Constant in astronomy) is that the speed of recession is L/T for every distance L the galaxy is located from you. These data are summarized in the table below.

Galaxy	Original distance of galaxy from galaxy A	Distance traveled by galaxy in time T as observed by galaxy A	Speed of recession of galaxy as observed by galaxy A	Speed of recession of galaxy as observed by galaxy A divided by original distance of galaxy from galaxy A
B	L	L	L/T	1/T {or (L/T)/L}
C	2L	2L	2L/T	1/T {or (2L/T)/(2L)}
D	3L	3L	3L/T	1/T {or (3L/T)/(3L)}

c. Do all galaxies see the same thing happening?

Yes. To a person on any galaxy, it appears that all the other galaxies are moving away from them and the speed of recession is proportional to the distance from your galaxy.



13. From the Associated Press dated 11/4/02: "Long Beach – Nearly 200 cars and big-rig trucks collided in two incidents on the fogbound Long Beach Freeway early yesterday, injuring dozens of people, nine critically, and closing the highway for hours… CHP officer Joseph Pace … said visibility was down to about 50 feet in heavy fog when the chain reaction crashes began just before 7 a.m … Some motorists estimated cars were moving at 25 to 35 mph."

From the Associated Press dated 11/5/02: "Los Angeles – The chain-reaction crashes that piled up nearly 200 cars on the Long Beach Freeway likely could have been avoided if drivers had simply slowed down when they hit foggy conditions, California Highway Patrol officers said yesterday. The crashes, which left a five-mile section of the freeway looking like an auto junkyard, shut down the highway for 11 hours Sunday. Eight people suffered critical or serious injuries in the accidents, which took place within minutes. Motorists reported driving into fog so thick it reminded some of being on an airliner as it travels into the clouds. "In that weather condition, we're sure if drivers had drastically reduced their speeds, this could have been avoided," said California Highway Patrol Officer Luis Mendoza."

a. What is the total breaking distance for a car traveling at 25 mph?
b. What is the total breaking distance for a car traveling at 35 mph?
c. How do these total breaking distances compare to the 50 foot visibility of that day? Why did the accidents occur?
d. What maximum speed should the cars have been traveling at if the visibility was only 50 feet.

a. $\frac{25 \text{ mi}}{\text{hr}} \times \frac{88\text{ft hr}}{\text{s} \quad 60 \text{ mi}} = \frac{36.7 \text{ ft}}{\text{s}}$

Reaction distance $= \frac{36.7 \text{ ft}}{\text{s}} \times 1.5 \text{ s} = 55 \text{ ft}$

Braking distance $= \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$

$$= \frac{36.7^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 39.6 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 55 ft + 39.6 ft = 94.6 ft.

b. a. $\frac{35\text{ mi}}{\text{hr}} \times \frac{88\text{ft hr}}{\text{s}\quad 60\text{ mi}} = \frac{51.3\text{ ft}}{\text{s}}$

Reaction distance $= \frac{51.3\text{ ft}}{\text{s}} \times 1.5\text{ s} = 77\text{ ft}$

Braking distance = $\frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$

$$= \frac{51.3^2\text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 77.4\text{ ft}$$

So stopping distance = reaction distance + braking distance = 77 ft + 77.4 ft = 154.4 ft.

c. The total stopping distances for cars traveling at 25 and 35 miles per hour is 94 and 154 feet respectively, much greater than the visibility of 50 feet. So the cars could not stop in time to avoid a crash.

d. The total stopping distance is equal to 50 ft. If you are initially traveling at a speed v, then the reaction distance is vt and the braking distance is $v^2/(2a)$.

.

So the total stopping distance (d_{total}) is:

$$d_{total} = vt + \frac{v^2}{2a}.$$

d_{total} = 50 ft, the braking deceleration $a = \frac{17\text{ ft}}{\text{s}^2}$, and the reaction time t = 1.5 s.

The equation that must be solved is a quadratic equation:

$$\frac{v^2}{2a} + vt - d_{total} = 0 \quad\text{or}\quad \frac{0.029\text{ s}^2 v^2}{\text{ft}} + 1.5\text{s } v - 50\text{ft} = 0$$

Using the quadratic formula

$$v = \frac{-1.5\text{ s} +/- \text{sqr}\{1.5^2\text{s}^2 - 4 * .029\text{s}^2/\text{ft} * (-50\text{ft})\}}{2 * 0.029\text{ s}^2/\text{ft}}$$

Only the + sign yields a physically meaningful solution, so:

$v = \frac{23\text{ ft}}{\text{s}}$ or 16 mph.

So if the visibility was only 50 feet, cars should have been traveling at a maximum speed of 16 mph.

14. From the January/February 2003 issue of the AAA magazine entitled “A Glaring Concern: HID headlights: boon to vehicular safety or blight on the automotive landscape?”

“Nevertheless, the visual improvement high-intensity discharge (HID) lamps provide is dramatic: A motorist using HIDs can see about 330 feet in front of the vehicle, compared with 190 feet with standard halogen lighting – almost a 75 percent improvement.”

“One of the problems with halogen lighting is that motorists often ‘overdrive’ their headlights … they’re unable to see people, animals, or objects until it’s too late.”

“The data are startling: Under perfect conditions (dry pavement, mechanically sound vehicle, antilock brakes, alert and skilled driver) at 45 mph, it takes 170 feet to stop your car; at 50 mph, 205 feet; and at 60 mph, 282 feet. So, somewhere between 45 and 50 mph, you’ve overdriven halogen headlights.”

a. What reaction time/breaking time is being assumed for the “perfect conditions” driver discussed above? Is this a fair discussion?

b. Discuss how HIDs can improve driving safety given your work so far in this unit. Consider the average driver with an average car and average reaction times.