Solutions:

Note that all the solutions involving unit conversions involve multiplying by different forms of one.

1. Convert 60 miles to the equivalent expression in feet. (1 mile=5280 feet).

$$60 \text{ miles} \times \frac{5280 \text{ ft}}{1 \text{ mile}} = 316{,}800 \text{ feet}$$

2. Convert 1 hour to the equivalent expression in s.

$$1 \cancel{\text{hour}} \times \frac{60 \cancel{\text{min}}}{1 \cancel{\text{hour}}} \times \frac{60 \text{ s}}{1 \cancel{\text{min}}} = 3600 \text{ s}$$

3. Convert $\frac{60 \text{ miles}}{\text{h}}$ to the equivalent expression in $\frac{\text{ft}}{\text{s}}$.

$$\frac{60 \cancel{\text{miles}}}{\cancel{\text{hour}}} \times \frac{5280 \text{ ft}}{1 \cancel{\text{mile}}} \times \frac{1 \cancel{\text{hour}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ s}} = \frac{88 \text{ ft}}{\text{s}}$$

4. Determine the number of s in a year.

$$\frac{60 \text{ s}}{\cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{\cancel{\text{h}}} \times \frac{24 \cancel{\text{h}}}{\cancel{\text{day}}} \times \frac{365.25 \cancel{\text{days}}}{\text{year}} = \frac{31{,}557{,}600 \text{ s}}{\text{year}}$$

5. Determine your height in meters. (Use 1 in = 2.54 cm)

Suppose you are 5 feet, 5 inches tall.

5 inches is $\frac{5}{12}$ of a foot. So 5 feet, 5 inches is $5\frac{5}{12}$ feet = 5.42 feet

$$5.42 \cancel{\text{ft}} \times \frac{12 \cancel{\text{in}}}{1 \cancel{\text{ft}}} \times \frac{2.54 \cancel{\text{cm}}}{1 \cancel{\text{in}}} \times \frac{1 \text{ m}}{100 \cancel{\text{cm}}} = 1.65 \text{ m}$$

6. Convert a length of 1 m to the equivalent length in cm, mm, µm, and km.

$$1\cancel{\text{m}} \times \frac{100 \text{ cm}}{1 \cancel{\text{m}}} = 100 \text{ cm}$$

$$1\cancel{\text{m}} \times \frac{1000 \text{ mm}}{1 \cancel{\text{m}}} = 1000 \text{ mm}$$

$$1\cancel{m} \times \frac{10^6\ \mu m}{1\ \cancel{m}} = 10^6\ \mu m$$

$$1\cancel{m} \times \frac{1\ km}{1000\ \cancel{m}} = \frac{1\ km}{1000} = 1 \times 10^{-3}\ km$$

7. Convert $\frac{1\ m}{s}$ to the equivalent expression in $\frac{km}{hr}$.

$$1\ \frac{\cancel{m}}{\cancel{s}} \times \frac{1km}{1000\ \cancel{m}} \times \frac{60\ \cancel{s}}{1\ \cancel{min}} \times \frac{60\ \cancel{min}}{1\ h} = \frac{3.6\ km}{h}$$

8. Convert $\frac{10\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{10\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{14.7\ ft}{s}$$

9. Convert $\frac{20\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{20\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{29.3\ ft}{s}$$

10. Convert $\frac{30\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{30\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{44\ ft}{s}$$

11. Convert $\frac{40\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{40\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{58.7ft}{s}$$

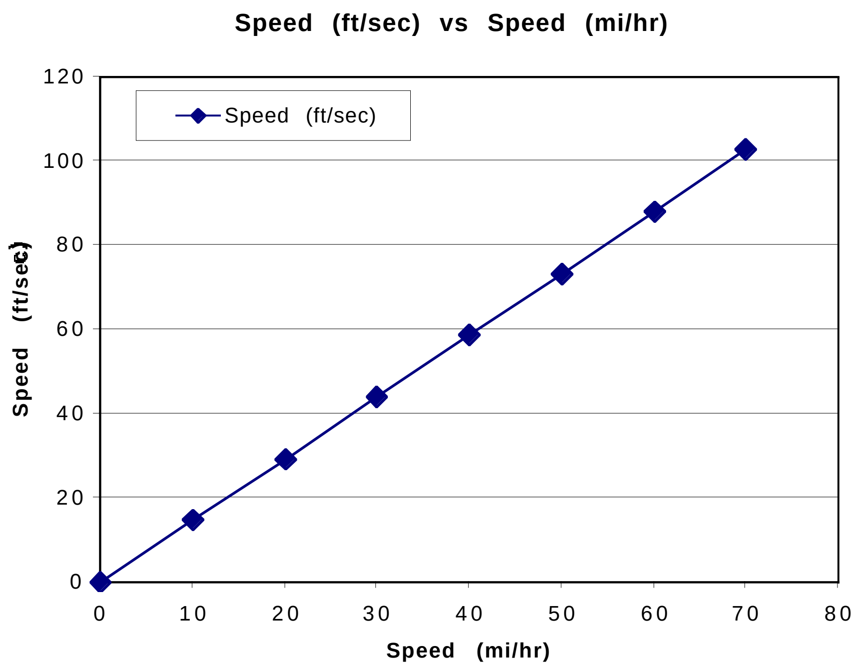
12. Convert $\frac{50\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{50\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{73.3\ ft}{s}$$

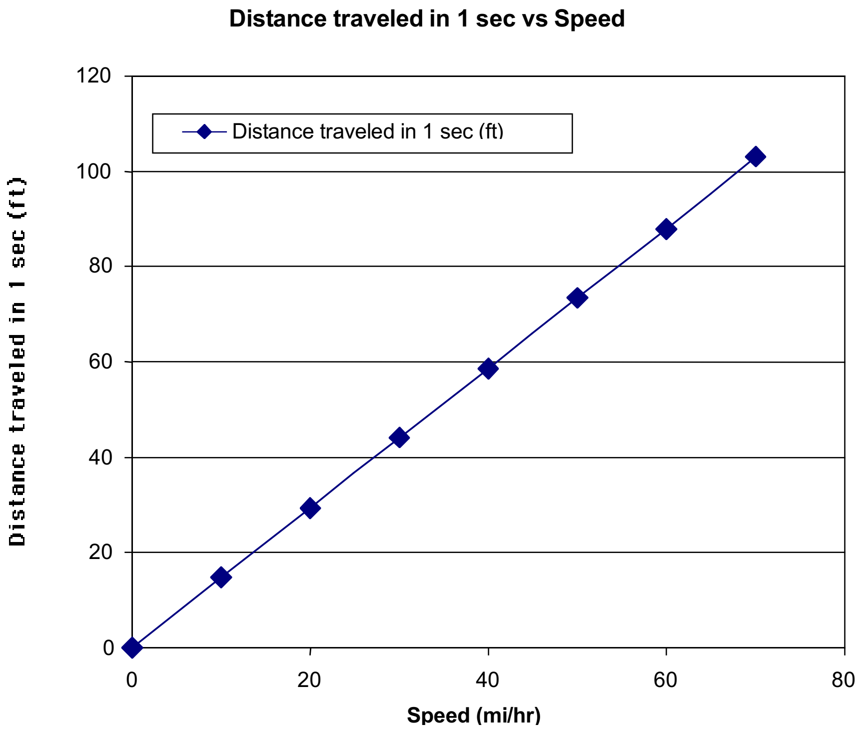
13. Convert $\frac{70 \text{ mi}}{\text{h}}$ to the equivalent expression in $\frac{\text{ft.}}{\text{s}}$

$$\frac{70 \cancel{\text{mi}}}{\cancel{\text{h}}} \times \frac{5280 \text{ ft}}{\cancel{\text{mi}}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}} = \frac{103 \text{ ft}}{\text{s}}$$

14.



15.


Distance traveled in 1 sec vs Speed
Distance traveled in 1 sec (ft)
Distance traveled in 1 sec (ft)
Speed (mi/hr)
0
20
40
60
80
100
120
0
20
40
60
80

16. Using dimensional analysis, determine a relationship between x, v, and t.

The units of x are m.
The units of v are $\frac{m}{s}$.

The units of t are s.

So $v\left(\frac{m}{s}\right) \sim \frac{x\,(m)}{t\,(s)}$ makes the dimensions match on each side of the equals sign.

So $v \sim \frac{x}{t}$

The equation x=vt relates the distance a car travels when it is moving at a speed v for a time t.

17. Using dimensional analysis, determine a relationship between x, v, and a.

The units of x are m.
The units of v are $\frac{m}{s}$.

The units of a are $\frac{m}{s^2}$.

So $a\left(\frac{m}{s^2}\right) \sim v^2 \frac{(m^2)}{(s^2)} \times \frac{1}{x\,(m)}$; so $a \sim \frac{v^2}{x}$ or $x \sim \frac{v^2}{a}$

In fact, the correct form of the equation can be obtained by isolating the variable that is squared and dividing that variable by 2:

$$ax = \frac{v^2}{2}$$

The distance traveled by a car traveling at speed v, braking at a constant deceleration a until it stops is:

$$x=\frac{v^2}{2a}$$

18. Using dimensional analysis, determine a relationship between v, a, and t.

The units of t are s.
The units of v are $\frac{m}{s}$.

The units of a are $\frac{m}{s^2}$.

So $\frac{a\ (m)}{(s^2)} \sim \frac{v\ (m)}{(s)} \times \frac{1}{t\ (s)}$; so $a \sim \frac{v}{t}$

The equation $a = \frac{v}{t}$ or $t = \frac{v}{a}$

relates the time t it takes for a car initially traveling at a speed v to stop if it undergoes constant deceleration a.

19. Using dimensional analysis, determine a relationship between x, a, and t.

The units of x are m.
The units of t are s.
The units of a are $\frac{m}{s^2}$.

So $a \frac{(m)}{(s^2)} \sim \frac{x\ (m)}{t^2\ (s^2)}$; so $a \sim \frac{x}{t^2}$

Again, the correct form of the equation can be obtained by isolating the variable that is squared and dividing that variable by 2:

$$\frac{x}{a} = \frac{t^2}{2}$$

$$x = \frac{at^2}{2}$$

relates the distance a car travels when accelerating from rest at a constant acceleration a for a time interval t.

20. Use the equations derived in problems 16 and 19 to determine the equation relating the distance traveled by an object during a time interval t that is initially traveling at a speed v_0, and then accelerates at a constant acceleration a for a time t.

From problem 16:
For constant speed: $x_f - x_i = v_o t$

From problem 19:
For constant acceleration: $x_f - x_i = \frac{at^2}{2}$

So for an object initially moving at constant speed that then accelerates with constant acceleration, we add the two equations above:

$$x_f - x_i = v_o t + \frac{at^2}{2}$$

21. Determine the distance traveled by a car initially at speed v_i that brakes at a constant deceleration a to a final speed of v_f. Use the relationship determined in problem 17.

From problem 17,

$$v_f^2 = 2ax_f$$

$$v_i^2 = 2ax_i$$

So subtracting the above 2 equations yields:

$$v_f^2 - v_i^2 = 2a(x_f - x_i)$$

22. Let's try to understand if the factor of _ that we put into the equation in problem 19 makes sense.

We will use the following equations:

(A) x=vt or v=x/t which means that x is the distance traveled by an object moving at a speed of v in a time interval t

and

(B) $a = (v_f - v_i)/t$ or $v_f = v_i + at$, which means that an object that initially is moving at speed v_i and accelerates at an acceleration of a for a time interval t ends up moving at speed v_f at the end of the time interval t.

Let's try to determine the position of an object that is initially at rest and accelerates at 1 m/s^2 for 10 s. Let's analyze this problem as 10 one second intervals and see what happens using the above equations. Let's assume that the distance traveled in each second is given by the speed at the beginning of the time interval times the length of the time interval – equation A above. Let's assume that the speed at the beginning of the next time interval can be determined by equation B above.

We can also use equation B above and consider t to be the total time the object has accelerated, in which case $v_i=0$.

a. Determine the distance the object moved between 0 and 1 s and the speed at which it is traveling after 1 s.
At the end of 1 s, the object moves a distance of x=vt=0m
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (1 s) = 1m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (1 s) = 1m/s

b. Determine the distance the object moved between 1 and 2 s and the speed at which it is traveling after 2 s.
At the end of 2 s, the object moves an additional distance of x=(1m/s)(1s) = 1 m
The speed at the beginning of the next interval is v_f = 1m/s + (1 m/s^2) (1 s) = 2m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (2 s) = 2m/s

c. Determine the distance the object moved between 2 and 3 s and the speed at which it is traveling after 3 s.
At the end of 3 s, the object moves an additional distance of x=(2m/s)(1s) = 2 m
The speed at the beginning of the next interval is v_f = 2m/s + (1 m/s^2) (1 s) = 3m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) 3 s) = 3m/s

d. Determine the distance the object moved between 3 and 4 s and the speed at which it is traveling after 4 s.
At the end of 4 s, the object moves an additional distance of x=(3m/s)(1s) = 3 m
The speed at the beginning of the next interval is v_f = 3m/s + (1 m/s^2) (1 s) = 4m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (4 s) = 4m/s

e. Determine the distance the object moved between 4 and 5 s and the speed at which it is traveling after 5 s.
At the end of 5 s, the object moves an additional distance of x=(4m/s)(1s) = 4 m
The speed at the beginning of the next interval is v_f = 4m/s + (1 m/s^2) (1 s) = 5m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (5 s) = 5m/s

f. Determine the distance the object moved between 5 and 6 s and the speed at which it is traveling after 6 s.
At the end of 6 s, the object moves an additional distance of x=(5m/s)(1s) = 5 m
The speed at the beginning of the next interval is v_f = 5m/s + (1 m/s^2) (1 s) = 6m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (6 s) = 6m/s

g. Determine the distance the object moved between 6 and 7 s and the speed at which it is traveling after 7 s.
At the end of 7 s, the object moves an additional distance of x=(6m/s)(1s) = 6 m
The speed at the beginning of the next interval is v_f = 6m/s + (1 m/s^2) (1 s) = 7m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (7 s) = 7m/s

h. Determine the distance the object moved between 7 and 8 s and the speed at which it is traveling after 8 s.
At the end of 8 s, the object moves an additional distance of x=(7m/s)(1s) = 7 m
The speed at the beginning of the next interval is v_f = 7m/s + (1 m/s^2) (1 s) = 8m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (8 s) = 8m/s

i. Determine the distance the object moved between 8 and 9 s and the speed at which it is traveling after 9 s.
At the end of 9 s, the object moves an additional distance of x=(8m/s)(1s) = 8 m
The speed at the beginning of the next interval is v_f = 8m/s + (1 m/s^2) (1 s) = 9m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (9 s) = 9m/s

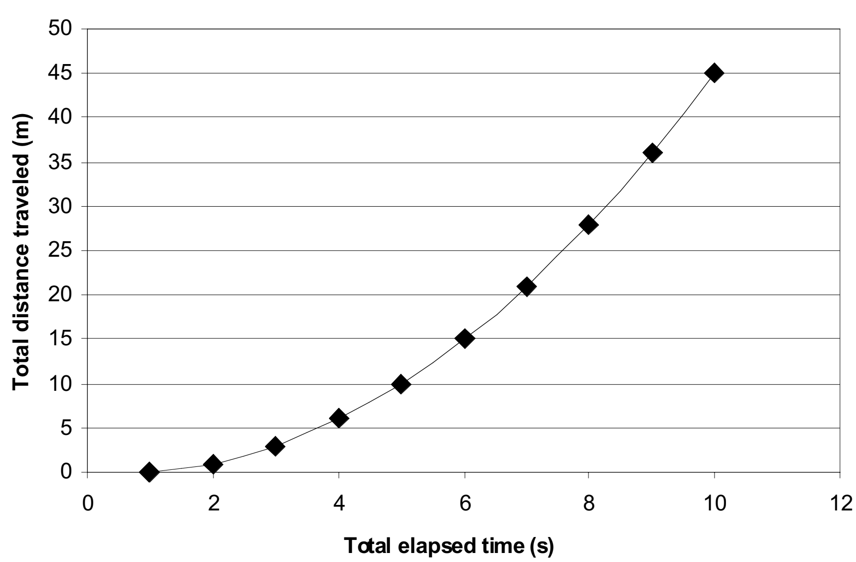
j. Determine the distance the object moved between 9 and 10 s and the speed at which it is traveling after 10 s.
At the end of 10 s, the object moves an additional distance of x=(9m/s)(1s) = 9 m
The speed at the beginning of the next interval is v_f = 9m/s + (1 m/s^2) (1 s) = 10m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (10 s)= 10m/s

k. Make a table that shows columns with time elapsed, the speed at the end of the time period, the additional distance traveled during the time interval, and the total distance traveled. Also determine $at^2/2$, at^2, $v_f^2/(2a)$, and v_f^2/a.

Time elapsed (s)	Speed at end of elapsed time (m/s)	Additional distance traveled (m)	Total distance traveled (m)	$x=\frac{at^2}{2}$	$x=at^2$	$\frac{v_f^2}{2a}$	$\frac{v_f^2}{a}$
1	1	0	0	0.5	1	0.5	1
2	2	1	1	2	4	2	4
3	3	2	3	4.5	9	4.5	9
4	4	3	6	8	16	8	16
5	5	4	10	12.5	25	12.5	25
6	6	5	15	18	36	18	36
7	7	6	21	24.5	49	24.5	49
8	8	7	28	32	64	32	64
9	9	8	36	40.5	81	40.5	81
10	10	9	45	50	100	50	100

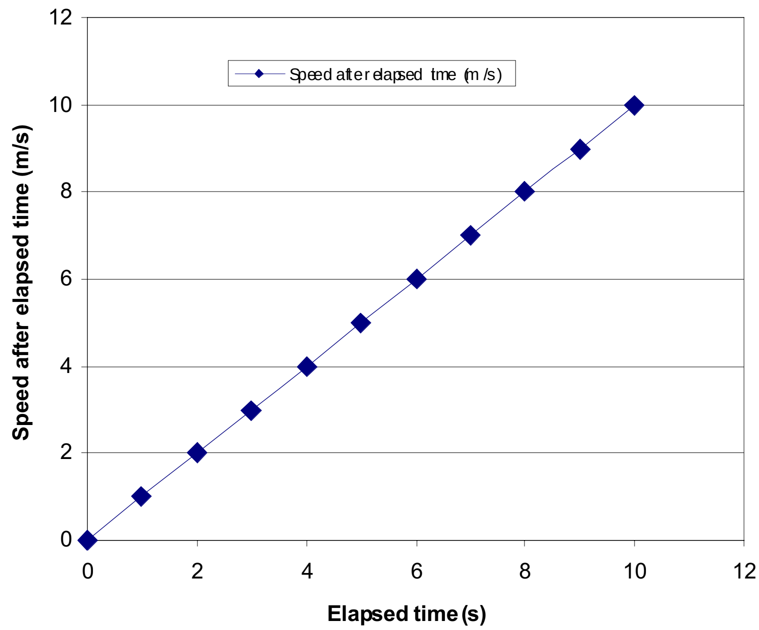
l. Is the total distance traveled closer to $at^2/2$ than at^2? Is the total distance traveled closer to $v_f^2/2a$ than v_f^2/a?
Yes. Yes.

m. Plot the total distance traveled in meters vs the total time elapsed:



n. How does the total distance traveled change with time during equal periods of constant accelerationi.
The graph shows that the total distance traveled rapidly increases with time during periods of constant acceleration.

o. Now plot the speed vs elapsed time.



p. How does the speed change for this constantly accelerating object.
The speed is increasing in a steady linear fashion, a characteristic of a constantly accelerating object.

q. Determine the total distance traveled by calculating the area under a speed vs time plot for this case of constant acceleration. It is just the area of a triangle with the base being the total elapsed time and the height being the final speed.

The elapsed time is 't' and the final speed is given by 'at'.

Distance traveled = 0.5 base * height

= 0.5 t at

$$x = \frac{at^2}{2}$$

r. Where did the factor of 1/2 come from?

Now we can finally see where the factor of 0.5 came from: it came from the equation for the area of a triangle.

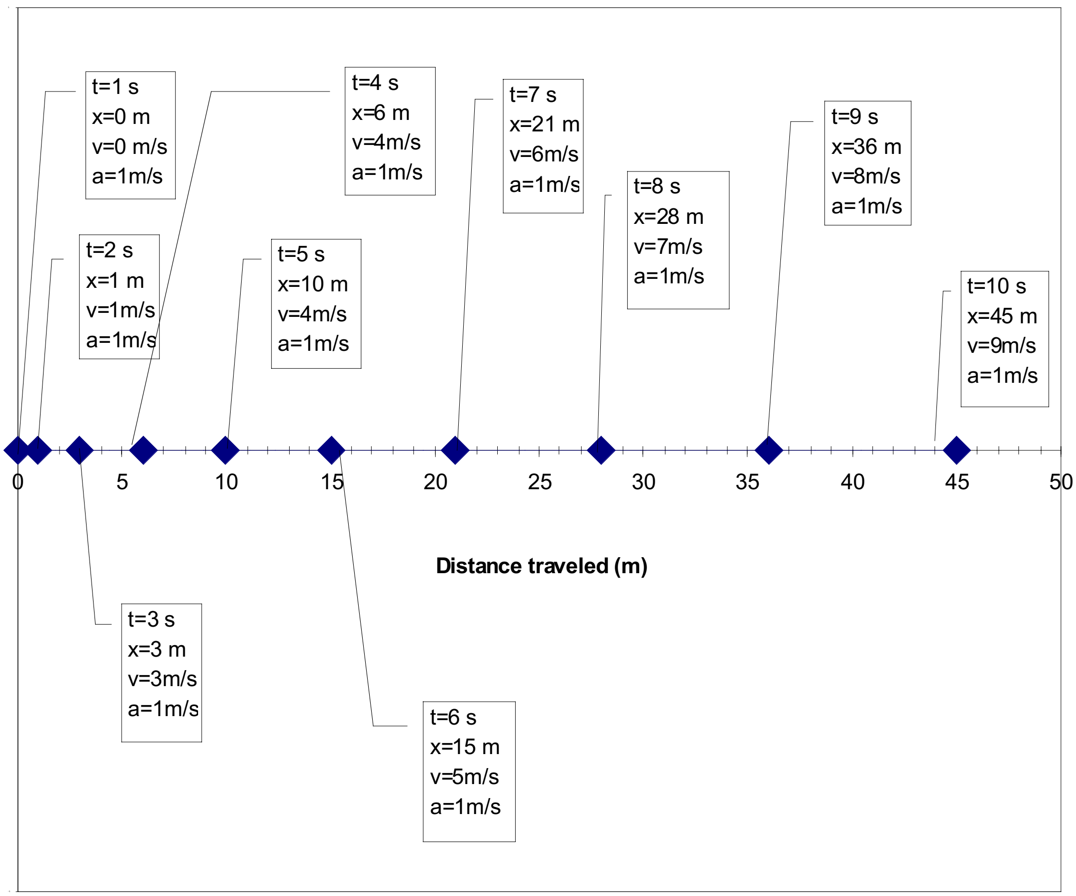
s. Note that since v_f =at, then t= v_f/a, the x-axis (time axis) can be written as v_f/a. The distance, which is the area under the curve of speed (v) vs time (v_f/a) curve, can be written how?

Distance traveled = 0.5 base * height

$$= 0.5 \frac{v_f}{2} * v_f$$

$$x = \frac{v_f^2}{2a}$$

t. On an axis showing the distance traveled in meters, show the time, position, speed, and acceleration after time intervals of 1 s from 0 to 10 s.



23. Suppose you are designing freeway on-ramps for the interstate highway system. How long should the on-ramp be if the speed limit on the freeway is:
a. 55 mph
b. 65 mph
c. 75 mph

Use $v^2 = 2ax$, since the car should accelerate to the speed limit so that it can merge safely. A typical car can accelerate to 60 mi/hr in about 10 s, corresponding to an acceleration of 60mi/hr/10 s = 88ft/s/10s = 8.8 ft/s^2. Let's assume a value of 8 ft/s^2.

a. $x = v^2/2a = \dfrac{(55\text{mi/hr} * 88\text{ft/s}/60\text{mi/hr})^2}{2*(8\text{ft/s}^2)} = 407$ ft.

b. $x = v^2/2a = \dfrac{(65\text{mi/hr} * 88\text{ft/s}/60\text{mi/hr})^2}{2*(8\text{ft/s}^2)} = 568$ ft.

c. $x = v^2/2a = \dfrac{(75\text{mi/hr} * 88\text{ft/s}/60\text{mi/hr})^2}{2*(8\text{ft/s}^2)} = 756$ ft.

According to the California Department of Transportation Manual (http://www.dot.ca.gov/hq/oppd/hdm/chaters/t504.htm)
a typical freeway distance for the acceleration lane is 330 m = 1083 ft.
The values determined by our calculation above are the minimum distance needed to accelerate, so the 1083 feet value includes a reasonable safety margin for a car to safely merge. A large truck accelerates much slower than this so truck drivers must be especially careful when they merge onto a freeway.

24. How do we understand an expression such as (10mi/hr)/s? It will become clearer in section 4. But let’s at least analyze it in the standard way we analyze fractions. Recall that dividing by a number is the same as multiplying by 1/number.

So: $(10\text{ mi/hr})/\text{s} = \dfrac{\frac{10\text{mi}}{\text{hr}}}{\text{s}} = \dfrac{10\text{ mi}}{\text{hr}} * \dfrac{1}{\text{s}} = \dfrac{10\text{ mi}}{\text{hr*s}}$

Similarly $(-10\text{m/s})/\text{s} = \dfrac{\frac{-10\text{m}}{\text{s}}}{\text{s}} = \dfrac{-10\text{ m}}{\text{s}} * \dfrac{1}{\text{s}} = \dfrac{-10\text{ m}}{\text{s*s}} = \dfrac{-10\text{ m}}{\text{s}^2}$

25. “How do you measure a year?”
Did you ever think that unit conversions could be the inspiration behind a hit Broadway musical? It’s true.
In the hit Broadway musical ‘Rent,’ there is a song called “Seasons of Love.”
The lyrics are shown below

ALL
Five hundred twenty-five thousand six hundred minutes
Five hundred twenty-five thousand moments so dear
Five hundred twenty-five thousand six hundred minutes
How do you measure - measure a year?

In daylights, in sunsets, in midnights, in cups of coffee
In inches, in miles, in laughter, in strife
In five hundred twenty-five thousand six hundred minutes
How do you measure a year in the life?

How about love? How about love?
How about love? Measure in love -
Seasons of love...
Seasons of love...

WOMAN
Five hundred twenty-five thousand six hundred minutes
Five hundred twenty-five thousand journeys to plan
Five hundred twenty-five thousand six hundred minutes
How do you measure the life of a woman or a man?

MAN
In truths that she learned or in times that he cried
In bridges he burned or the way that she died

ALL
It's time now to sing out though the story never ends
Let's celebrate, remember year in the life of friends
Remember the love... Remember the love...
Remember the love... Measure in love -

WOMAN
Measure, measure your life in love...

ALL
Seasons of love...
Seasons of love...

a. Show that there are 525,600 minutes in a year.
b. How many daylights are there in a year?
c. How many sunsets are there in a year?
d. How many midnights are there in a year?
e. How many cups of coffee do you drink in a year?
f. The earth is 93,000,000 miles from the sun. How many inches does the earth travel around the sun in a year?
g. How many miles does the earth travel around the sun in a year?
h. How many times do you laugh in a year?

i. How many times do you have strife in your life in a year?
j. Write some lyrics to a song that includes unit conversions.

Answers:

a. Show that there are 525,600 minutes in a year.

$$\frac{60\text{ min}}{\cancel{\text{h}}} \times \frac{24\ \cancel{\text{h}}}{\cancel{\text{day}}} \times \frac{365\ \cancel{\text{days}}}{\text{year}} = \frac{525{,}600\text{ min}}{\text{year}}$$

b. How many daylights are there in a year?

$$\frac{1\text{ daylight}}{\text{day}} \times \frac{365\text{ days}}{\text{year}} = \frac{365\text{ daylights}}{\text{year}}$$

c. How many sunsets are there in a year?

$$\frac{1\text{ sunset}}{\text{day}} \times \frac{365\text{ days}}{\text{year}} = \frac{365\text{ sunsets}}{\text{year}}$$

d. How many midnights are there in a year?

$$\frac{1\text{ midnight}}{\text{day}} \times \frac{365\text{ days}}{\text{year}} = \frac{365\text{ midnights}}{\text{year}}$$

e. How many cups of coffee do you drink in a year?

$$\frac{2\text{ cups}}{\text{day}} \times \frac{365\text{ days}}{\text{year}} = \frac{730\text{ cups}}{\text{year}}$$

f. The earth is 93,000,000 miles from the sun. How many inches does the earth travel around the sun in a year?

The earth's orbit is nearly circular, the distance traveled is the circumference of the orbit, and the circumference of a circle is 2 x pi x radius, so:

$$\text{Distance} = 2 \times 3.14 \times 93 \times 10^{6}\text{ miles} \times \frac{5280\text{ ft}}{\text{mile}} \times \frac{12\text{ in}}{\text{ft}} = 3.70 \times 10^{13}\text{ inches}$$

g. How many miles does the earth travel around the sun in a year?

The earth's orbit is nearly circular, ,the distance traveled is the circumference of the orbit, and the circumference of a circle is 2 x pi x radius, so:

$$\text{Distance} = 2 \times 3.14 \times 93 \times 10^{6}\text{ miles} = 5.84 \times 10^{8}\text{ miles}$$

h. How many times do you laugh in a year?

$$\frac{20\text{ laughs}}{\text{day}} \times \frac{365\text{ days}}{\text{year}} = \frac{7300\text{ laughs}}{\text{year}}$$

i. How many times do you have strife in your life in a year?

$$\frac{1\text{ strife}}{\text{week}} \times \frac{52\text{ weeks}}{\text{year}} = \frac{52\text{ strifes}}{\text{year}}$$

j. Write some lyrics to a song that includes unit conversions.

Good luck and have fun. Perform it in front of the class for extra enjoyment!

26. Hydroplaning can occur when driving on roads on which about 0.2 inch of water has accumulated. Suppose there is a heavy downpour where it is raining at a rate of 4 inches per hour and there is depression on the road that does not drain. All of the water in this depression does however get splashed away when a car drives over it. (This problem is based on a crash that happened to a friend of mine.)

a. How long will it take for the depression to fill to the point where it will lead to hydroplaning?

$$\text{Time} = \frac{0.20\text{ in}}{4\text{ in/hr}} = 0.05\text{ hr} \times \frac{60\text{min}}{1\text{ hr}} = 3\text{ min}$$

b. Suppose that you are driving in the car pool (diamond) lane and a car was in that lane 2 minutes ago. Will you hydroplane and probably lose control of your car?

No. The water in the depression will have accumulated to a depth of less than 0.2 inches so your car will not hydroplane.

c. Suppose that you are driving in the car pool (diamond) lane and no other car has used that lane for the last 4 minutes. Will you hydroplane and probably lose control of your car?

Yes. The water in the depression will have accumulated to a depth of greater than 0.2 inches so your car will hydroplane.

d. How will the above calculations affect how you drive in the rain?

If there are not many cars driving in my lane, I should slow down a lot because it is likely that I will hydroplane in areas of the road where there are small depressions. It is best if I drive in the lanes that are most used, not ones that are hardly used at all.