Staying Alive: The Physics, Mathematics, and Engineering of Safe Driving

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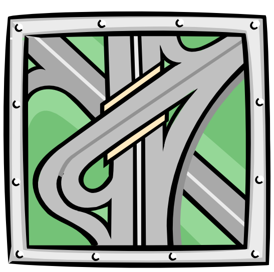
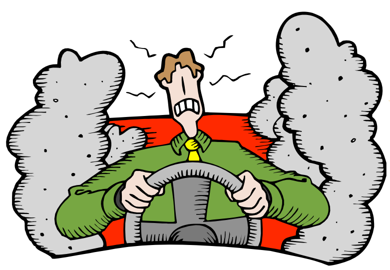
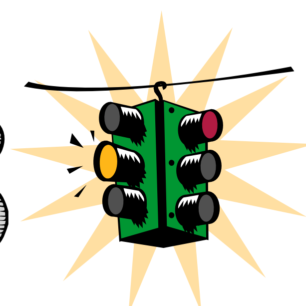
December 16, 2008

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Introduction

For most teenagers, I suspect that mechanics is a fairly boring introduction to the field of physics. In my brief exposure to tutoring high school students and in perusing introductory physics books, it seems that most student problems and activities are very dull. It is hard to get excited about position, velocity, and acceleration, especially if they appear rather irrelevant to the average teenager's life. Mechanics, the first (and often dominant) topic that students study in a physics class, usually involves problems such as how long thrown balls stay up in the air or the trajectory of cannon balls. Most teenagers are interested in themselves, travelling, or cars: therefore, this module revolves around these subjects.

An introductory section emphasizes the importance of units, an important topic often ignored, as well as dimensional analysis. This topic is critical for understanding the topic of auto mechanics as well as other fields of science. This unit purposely uses units of feet as well as meters so that students gain experience in using and converting units.

The module is comprised of three projects. In the first project, students learn about position and movement by moving themselves and measuring their position and speed. After students master the basic concepts of position, distance, and speed, they will plan a trip using a map for their second project. Most students like to travel and mastering the concepts of position, time, speed, and velocity will provide them will allow them to plan trips, including how long it will take them to be driven to their sports practice at a new field, or where they should plan on spending the night during their next trip.

The third and major part of the module was written in an attempt to save lives. Too many teenage drivers get into crashes. I hope that by providing students with some realization of how quickly things can happen while they are driving that they might drive more prudently and defensively. According to an article in the 2/4/02 issue of Design News, "virtually all automakers around the world agree that the vast majority of accidents are caused by driver inattentiveness, and that a single second of warning could yield astonishing changes in the frequency and severity of collisions." In this section, students learn about braking distances, being drunk or talking on a cell phone while driving, the dangers of driving at night, passing on 2-lane roads, reaching for a CD while driving, and how much space to leave between cars on a freeway. Some fun applications of mechanics, such as the Doppler effect, gas mileage policy, and the expansion of the universe, are also covered.

This need to complete this unit was catalyzed, in part, by an editorial (Safe cars, great; safe drivers, better (5/1/00)) and a subsequent letter to the editor in Design News magazine. That letter to the editor by Steve Buchholz in the 7/3/00 issue stated "I was hoping to hear the voice of an engineer calling for programs to help those who take on the responsibility of controlling a vehicle and the safety of others to understand the physical laws under which they must operate, and programs to help people understand the limitations of their vehicles … and their driving skills." A similar sentiment is expressed by Leonard Evans in his excellent book, Traffic Safety and the Driver.

Note that I have not tried to reproduce the typical high school mechanics text nor have I attempted to derive standard mechanics equations. This unit will hopefully be useful to introduce, complement, or supplement existing instructional materials.

Most jobs require summarizing and presenting data in a meaningful manner, such as in tables, spreadsheets, and graphs. Therefore, many of these investigations involve calculations for a wide range of parameters, with subsequent tabulating and plotting of the data.

This module is at an early stage of development, with much more yet to come. It is still incomplete in some areas.

I have tried to write this module so that it may be used at either the middle school or high school level. Please let me know whether or not I have succeeded. Constructive suggestions are always welcome - send me an email at Larry.Woolf@gat.com.

Dedication

This unit is dedicated to the memory of Chad Hama, age 19, who died in a tragic traffic crash during the development of this module.

Investigation #1: The Power of (the number) ONE

[Note: the Power of One is a commonly used phrase in advertising and politics, so I thought it would be easy for students to remember. Please do not confuse this title with anything to do with taking a number and raising it to the first power.]

The power of the number one is vastly unappreciated. Its use forms the basis for using and transforming units of measurement – a vitally important concept in science.

Why are units important?

Ask your students what are some reasons for using units.

There are two reasons.

First, the value of a parameter without its corresponding unit is meaningless. Consider the following statements without units:

1. The water temperature is 32.
2. It's about 20 from here.
3. I'll call you back in about 2.

These statements are either unclear or require you to assume a unit of measurement.

They have meaning only when units are included.

1. The water temperature is 32 degrees Celsius.
2. It's about 20 kilometers from here.
3. I'll call you back in about 2 minutes.

If units are not included, the reader must infer the units that you are using. And their inference may be wrong - with serious consequences. In 1999, a confusion between metric and English units caused the loss of the $125 million Mars Climate Orbiter spacecraft.

Consider also this news story dated 11/2/02 from the New York Times News Service::

"Dublin, Ireland – The country will convert all its directional road signs and speed limits to the metric system by the spring of 2004 in compliance with European Union rules, Transport Minister Seamus Brennan said. Speed limits are currently stated in miles per hour, but signs showing distances are notorious for confusing travelers by varying in whether they display distances in miles or kilometers and by not indicating which units are used."

In a 12/20/99 letter to the editor of Design News magazine, Kevin Acheson, Chief Engineer of The Gear Works put it this way, "A number without units is meaningless. In high school I had a Physics teacher who constantly was harping on units. The only way you could pass her class was to show the units with the formulas, as well as with the final answer. At the time it seemed silly, but I did what she required to I could pass. Few things in my education have served me better than the lesson she drove how about always using units. In the 18+ years I have been doing engineering, I have seen more and more young engineers who haven't learned how easy it is to think you have a correct answer only to forget to apply some unit conversion. If anyone thinks that universally switching to the metric system will eliminate stupid errors, they are sadly mistaken. Is a meter the same as a kilometer? Of course not! Without units, no one knows what a number really means.

The second reason is that units provide us with an independent check of the process by which we calculate the answer to our problem; this is called "dimensional analysis." If the units (and magnitude) of the final answer do not make sense, then we know the answer is incorrect. We can also determine the functional relationship between different parameters using dimensional analysis.

Before exploring this aspect of units, let's come back to the power of one.
We know that 1 minute = 60 s, or

$$1 \text{ min} = 60 \text{ s}. \qquad (1)$$

Dividing both sides of equation (1) by 60 s yields an expression (actually an identity) for 1:

$$\frac{1 \text{ min}}{60\text{s}} = 1 \qquad (2)$$

Dividing both sides of the equation (1) by 1 minute yields a complementary expression for 1:

$$\frac{60 \text{ s}}{1 \text{ min}} = 1 \qquad (3)$$

Multiplying or dividing by one does not change the value of a number. So we can always multiply (or divide) by one and not affect the value of the number. If we have a value for some time in seconds and we want to convert it to minutes, we multiply the value by the appropriate expression for one. The procedure is:

A. if you want to convert a unit in the numerator, multiply by the expression for one that has that unit in the denominator.
B. if you want to convert a unit in the denominator, multiply by the expression for one that has that unit in the numerator.

Here's one other major piece of advice to assist in keeping track of units:
ALWAYS USE HORIZONTAL LINES TO SEPARATE FRACTIONS.
(NEVER USE DIAGONAL SLASHES.)

For example:

Convert 120 s to its value in minutes.

$$120 \text{ s} = \frac{120 \text{ s}}{1}$$

We want to convert the unit of s, which is in the numerator, so rule A tells us to use the expression for 1 that has the unit of s in the denominator:

$$\frac{1 \text{ min}}{60 \text{ s}} = 1$$

So:

$$120 \text{ s} = \frac{120 \text{ s} \times 1 \text{ min}}{60 \text{ s}};$$ since 1 min x 1 s = 1 s x 1 min, then

$$120\text{ s} = \frac{120\text{ min} \times 1\text{ s}}{60\text{ s}}$$

$$= \frac{120\text{ min} \times 1\text{ s}}{60 \qquad \text{s}} \quad \text{Since } \frac{1\text{ s}}{1\text{ s}} = 1\text{, then}$$

120 s = 2 min.

Problems:

1. Convert 60 miles to the equivalent expression in feet. (1 mile=5280 feet)
2. Convert 1 hour to the equivalent expression in seconds.
3. Convert $\frac{60 \text{ miles}}{h}$ to the equivalent expression in $\frac{ft}{s}$.

4. Determine the number of seconds in a year.
5. Determine your height in meters. (Use 1 in = 2.54 cm)
6. Convert a length of 1 m to the equivalent length in cm, mm, μm, and km.
7. Convert $\frac{1 \text{ m}}{s}$ to the equivalent expression in $\frac{km}{h}$.

8. Convert $\frac{10 \text{ mi}}{h}$ to the equivalent expression in $\frac{ft}{s}$.

9. Convert $\frac{20 \text{ mi}}{h}$ to the equivalent expression in $\frac{ft}{s}$.

10. Convert $\frac{30 \text{ mi}}{h}$ to the equivalent expression in $\frac{ft}{s}$.

11. Convert $\frac{40 \text{ mi}}{h}$ to the equivalent expression in $\frac{ft}{s}$.

12. Convert $\frac{50 \text{ mi}}{h}$ to the equivalent expression in $\frac{ft}{s}$.

13. Convert $\frac{70 \text{ mi}}{h}$ to the equivalent expression in $\frac{ft}{s}$.

14. Make a graph of speed in feet per second vs. speed in miles per hour based on the above conversions.

15. Make a graph of distance traveled in feet in 1 s vs speed in mph.

16. Using dimensional analysis, determine a relationship between x, v, and t. To do this, write an equation with these variables so that the dimensions match on each side of the equals sign.

17. Using dimensional analysis, determine a relationship between x, v, and a. To do this, write an equation with these variables so that the dimensions match on each side of the equals sign.

18. Using dimensional analysis, determine a relationship between v, a, and t. To do this, write an equation with these variables so that the dimensions match on each side of the equals sign.

19. Using dimensional analysis, determine a relationship between x, a, and t. To do this, write an equation with these variables so that the dimensions match on each side of the equals sign.

20. Use the equations derived in problems 16 and 19 to determine the equation relating the distance traveled by an object initially traveling at a speed v_0 and then accelerating at a constant acceleration a for a time t.

21. Determine the distance traveled by a car initially at speed v_i that brakes at a constant deceleration a to a final speed of v_f. Use the relationship determined in problem 17.

Solutions:

Note that all the solutions involving unit conversions involve multiplying by different forms of one.

1. Convert 60 miles to the equivalent expression in feet. (1 mile=5280 feet).

$$60 \text{ miles} \times \frac{5280 \text{ ft}}{1 \text{ mile}} = 316{,}800 \text{ feet}$$

2. Convert 1 hour to the equivalent expression in s.

$$1\ \cancel{\text{hour}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hour}}} \times \frac{60 \text{ s}}{1\ \cancel{\text{min}}} = 3600 \text{ s}$$

3. Convert $\frac{60 \text{ miles}}{\text{h}}$ to the equivalent expression in $\frac{\text{ft}}{\text{s}}$.

$$\frac{60\ \cancel{\text{miles}}}{\cancel{\text{hour}}} \times \frac{5280 \text{ ft}}{1\ \cancel{\text{mile}}} \times \frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60 \text{ s}} = \frac{88 \text{ ft}}{\text{s}}$$

4. Determine the number of s in a year.

$$\frac{60 \text{ s}}{\cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{\cancel{\text{h}}} \times \frac{24\ \cancel{\text{h}}}{\cancel{\text{day}}} \times \frac{365.25\ \cancel{\text{days}}}{\text{year}} = \frac{31{,}557{,}600 \text{ s}}{\text{year}}$$

5. Determine your height in meters. (Use 1 in = 2.54 cm)

Suppose you are 5 feet, 5 inches tall.

5 inches is $\frac{5}{12}$ of a foot. So 5 feet, 5 inches is $5\frac{5}{12}$ feet = 5.42 feet

$$5.42\ \cancel{\text{ft}} \times \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{1 \text{ m}}{100\ \cancel{\text{cm}}} = 1.65 \text{ m}$$

6. Convert a length of 1 m to the equivalent length in cm, mm, μm, and km.

$$1\cancel{\text{m}} \times \frac{100 \text{ cm}}{1\ \cancel{\text{m}}} = 100 \text{ cm}$$

$$1\cancel{\text{m}} \times \frac{1000 \text{ mm}}{1\ \cancel{\text{m}}} = 1000 \text{ mm}$$

$$1\cancel{m} \times \frac{10^6\ \mu m}{1\ \cancel{m}} = 10^6\ \mu m$$

$$1\cancel{m} \times \frac{1\ km}{1000\ \cancel{m}} = \frac{1\ km}{1000} = 1 \times 10^{-3}\ km$$

7. Convert $\frac{1\ m}{s}$ to the equivalent expression in $\frac{km}{hr}$.

$$\frac{1\ \cancel{m}}{\cancel{s}} \times \frac{1km}{1000\ \cancel{m}} \times \frac{60\ \cancel{s}}{1\ \cancel{min}} \times \frac{60\ \cancel{min}}{1\ h} = \frac{3.6\ km}{h}$$

8. Convert $\frac{10\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{10\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{14.7\ ft}{s}$$

9. Convert $\frac{20\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{20\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{29.3\ ft}{s}$$

10. Convert $\frac{30\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{30\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{44\ ft}{s}$$

11. Convert $\frac{40\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{40\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{58.7ft}{s}$$

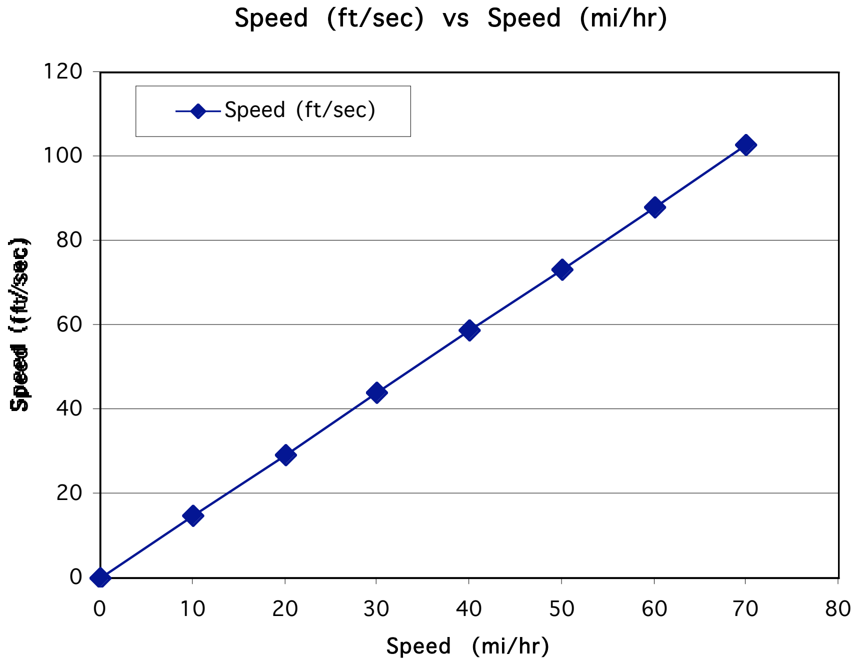
12. Convert $\frac{50\ mi}{h}$ to the equivalent expression in $\frac{ft.}{s}$

$$\frac{50\ \cancel{mi}}{\cancel{h}} \times \frac{5280\ ft}{\cancel{mi}} \times \frac{1\ \cancel{h}}{3600\ s} = \frac{73.3\ ft}{s}$$

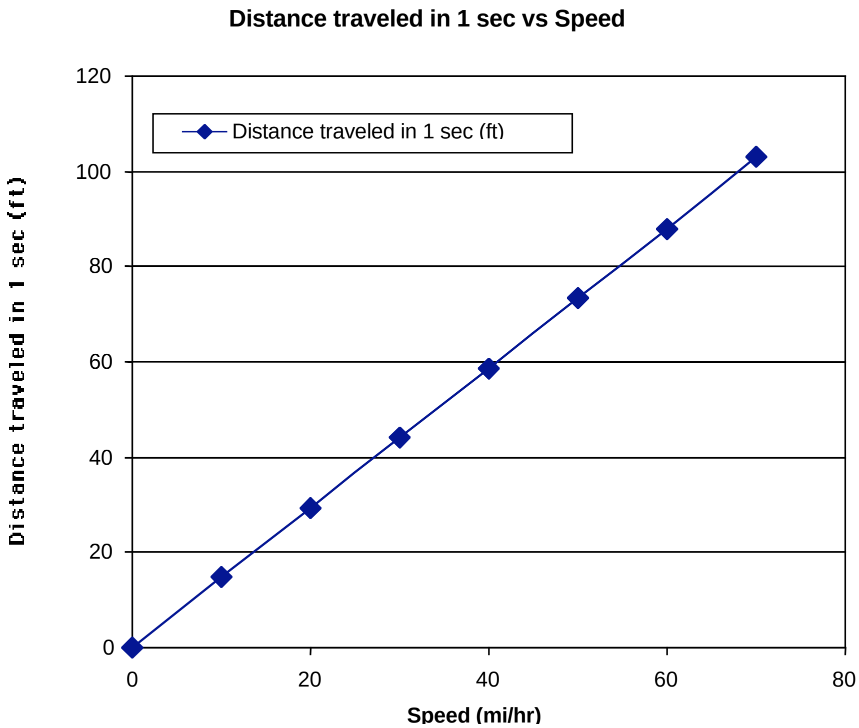
13. Convert $\frac{70 \text{ mi}}{\text{h}}$ to the equivalent expression in $\frac{\text{ft.}}{\text{s}}$

$$\frac{70 \not{\text{mi}}}{\not{\text{h}}} \times \frac{5280 \text{ ft}}{\not{\text{mi}}} \times \frac{1 \not{\text{h}}}{3600 \text{ s}} = \frac{103 \text{ ft}}{\text{s}}$$

14.



15.



Distance traveled in 1 sec vs Speed
Distance traveled in 1 sec (ft)
Distance traveled in 1 sec (ft)
Speed (mi/hr)
0
20
40
60
80
100
120

16. Using dimensional analysis, determine a relationship between x, v, and t.

The units of x are m.
The units of v are $\frac{m}{s}$.

The units of t are s.

So $v \left(\frac{m}{s}\right) \sim \frac{x\,(m)}{t\,(s)}$ makes the dimensions match on each side of the equals sign.

So $v \sim \frac{x}{t}$

The equation x=vt relates the distance a car travels when it is moving at a speed v for a time t.

17. Using dimensional analysis, determine a relationship between x, v, and a.

The units of x are m.
The units of v are $\frac{m}{s}$.

The units of a are $\frac{m}{s^2}$.

So $a \left(\frac{m}{s^2}\right) \sim v^2 \frac{(m^2)}{(s^2)} \times \frac{1}{x\,(m)}$; so $a \sim \frac{v^2}{x}$ or $x \sim \frac{v^2}{a}$

In fact, the correct form of the equation can be obtained by isolating the variable that is squared and dividing that variable by 2:

$$ax = \frac{v^2}{2}$$

The distance traveled by a car traveling at speed v, braking at a constant deceleration a until it stops is:

$$x=\frac{v^2}{2a}$$

18. Using dimensional analysis, determine a relationship between v, a, and t.

The units of t are s.
The units of v are $\frac{m}{s}$.

The units of a are $\frac{m}{s^2}$.

So $\frac{a\ (m)}{(s^2)} \sim \frac{v\ (m)}{(s)} \times \frac{1}{t\ (s)}$; so $a \sim \frac{v}{t}$

The equation $a = \frac{v}{t}$ or $t = \frac{v}{a}$

relates the time t it takes for a car initially traveling at a speed v to stop if it undergoes constant deceleration a.

19. Using dimensional analysis, determine a relationship between x, a, and t.

The units of x are m.
The units of t are s.
The units of a are $\frac{m}{s^2}$.

So $a \frac{(m)}{(s^2)} \sim \frac{x\ (m)}{t^2\ (s^2)}$; so $a \sim \frac{x}{t^2}$

Again, the correct form of the equation can be obtained by isolating the variable that is squared and dividing that variable by 2:

$$\frac{x}{a} = \frac{t^2}{2}$$

$$x = \frac{at^2}{2}$$

relates the distance a car travels when accelerating from rest at a constant acceleration a for a time interval t.

20. Use the equations derived in problems 16 and 19 to determine the equation relating the distance traveled by an object during a time interval t that is initially traveling at a speed v_0, and then accelerates at a constant acceleration a for a time t.

From problem 16:
For constant speed: $x_f - x_i = v_o t$

From problem 19:
For constant acceleration: $x_f - x_i = \frac{at^2}{2}$

So for an object initially moving at constant speed that then accelerates with constant acceleration, we add the two equations above:

$$x_f - x_i = v_o t + \frac{at^2}{2}$$

21. Determine the distance traveled by a car initially at speed v_i that brakes at a constant deceleration a to a final speed of v_f. Use the relationship determined in problem 17.

From problem 17,

$$v_f^2 = 2ax_f$$

$$v_i^2 = 2ax_i$$

So subtracting the above 2 equations yields:

$$v_f^2 - v_i^2 = 2a(x_f - x_i)$$

22. Let's try to understand if the factor of ½ that we put into the equation in problem 19 makes sense.

We will use the following equations:

(A) $x=vt$ or $v=x/t$ which means that x is the distance traveled by an object moving at a speed of v in a time interval t

and

(B) $a = (v_f-v_i)/t$ or $v_f = v_i + at$, which means that an object that initially is moving at speed v_i and accelerates at an acceleration of a for a time interval t ends up moving at speed v_f at the end of the time interval t.

Let's try to determine the position of an object that is initially at rest and accelerates at 1 $m/s^2$for 10 s. Let's analyze this problem as 10 one second intervals and see what happens using the above equations. Let's assume that the distance traveled in each second is given by the speed at the beginning of the time interval times the length of the time interval – equation A above. Let's assume that the speed at the beginning of the next time interval can be determined by equation B above.

We can also use equation B above and consider t to be the total time the object has accelerated, in which case $v_i=0$.

a. Determine the distance the object moved between 0 and 1 s and the speed at which it is traveling after 1 s.
At the end of 1 s, the object moves a distance of $x=vt=0m$
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (1 s) = 1m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (1 s) = 1m/s

b. Determine the distance the object moved between 1 and 2 s and the speed at which it is traveling after 2 s.
At the end of 2 s, the object moves an additional distance of x=(1m/s)(1s) = 1 m
The speed at the beginning of the next interval is v_f = 1m/s + (1 m/s^2) (1 s) = 2m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (2 s) = 2m/s

c. Determine the distance the object moved between 2 and 3 s and the speed at which it is traveling after 3 s.
At the end of 3 s, the object moves an additional distance of x=(2m/s)(1s) = 2 m
The speed at the beginning of the next interval is v_f = 2m/s + (1 m/s^2) (1 s) = 3m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) 3 s) = 3m/s

d. Determine the distance the object moved between 3 and 4 s and the speed at which it is traveling after 4 s.
At the end of 4 s, the object moves an additional distance of x=(3m/s)(1s) = 3 m
The speed at the beginning of the next interval is v_f = 3m/s + (1 m/s^2) (1 s) = 4m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (4 s) = 4m/s

e. Determine the distance the object moved between 4 and 5 s and the speed at which it is traveling after 5 s.
At the end of 5 s, the object moves an additional distance of x=(4m/s)(1s) = 4 m
The speed at the beginning of the next interval is v_f = 4m/s + (1 m/s^2) (1 s) = 5m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (5 s) = 5m/s

f. Determine the distance the object moved between 5 and 6 s and the speed at which it is traveling after 6 s.
At the end of 6 s, the object moves an additional distance of x=(5m/s)(1s) = 5 m
The speed at the beginning of the next interval is v_f = 5m/s + (1 m/s^2) (1 s) = 6m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (6 s) = 6m/s

g. Determine the distance the object moved between 6 and 7 s and the speed at which it is traveling after 7 s.
At the end of 7 s, the object moves an additional distance of x=(6m/s)(1s) = 6 m
The speed at the beginning of the next interval is v_f = 6m/s + (1 m/s^2) (1 s) = 7m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (7 s) = 7m/s

h. Determine the distance the object moved between 7 and 8 s and the speed at which it is traveling after 8 s.
At the end of 8 s, the object moves an additional distance of x=(7m/s)(1s) = 7 m
The speed at the beginning of the next interval is v_f = 7m/s + (1 m/s^2) (1 s) = 8m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (8 s) = 8m/s

i. Determine the distance the object moved between 8 and 9 s and the speed at which it is traveling after 9 s.
At the end of 9 s, the object moves an additional distance of x=(8m/s)(1s) = 8 m
The speed at the beginning of the next interval is v_f = 8m/s + (1 m/s^2) (1 s) = 9m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (9 s) = 9m/s

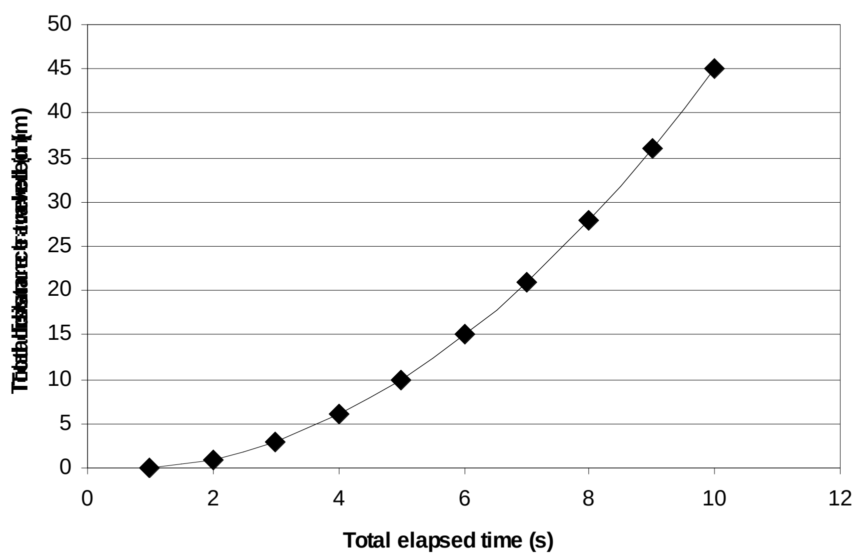
j. Determine the distance the object moved between 9 and 10 s and the speed at which it is traveling after 10 s.
At the end of 10 s, the object moves an additional distance of x=(9m/s)(1s) = 9 m
The speed at the beginning of the next interval is v_f = 9m/s + (1 m/s^2) (1 s) = 10m/s
The speed at the beginning of the next interval is v_f = 0m/s + (1 m/s^2) (10 s)= 10m/s

k. Make a table that shows columns with time elapsed, the speed at the end of the time period, the additional distance traveled during the time interval, and the total distance traveled. Also determine $at^2/2$, at^2, $v_f^2/(2a)$, and v_f^2/a.

Time elapsed (s)	Speed at end of elapsed time (m/s)	Additional distance traveled (m)	Total distance traveled (m)	$x=\frac{at^2}{2}$	$x=at^2$	$\frac{v_f^2}{2a}$	$\frac{v_f^2}{a}$
1	1	0	0	0.5	1	0.5	1
2	2	1	1	2	4	2	4
3	3	2	3	4.5	9	4.5	9
4	4	3	6	8	16	8	16
5	5	4	10	12.5	25	12.5	25
6	6	5	15	18	36	18	36
7	7	6	21	24.5	49	24.5	49
8	8	7	28	32	64	32	64
9	9	8	36	40.5	81	40.5	81
10	10	9	45	50	100	50	100

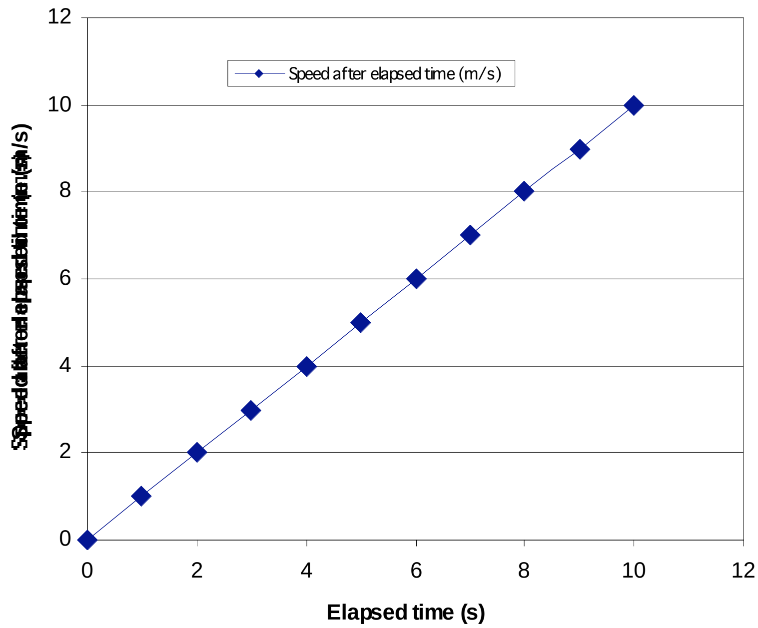
l. Is the total distance traveled closer to $at^2/2$ than at^2? Is the total distance traveled closer to $v_f^2/2a$ than v_f^2/a?
Yes. Yes.

m. Plot the total distance traveled in meters vs the total time elapsed:



n. How does the total distance traveled change with time during equal periods of constant accelerationi.
The graph shows that the total distance traveled rapidly increases with time during periods of constant acceleration.

o. Now plot the speed vs elapsed time.



p. How does the speed change for this constantly accelerating object.
The speed is increasing in a steady linear fashion, a characteristic of a constantly accelerating object.

q. Determine the total distance traveled by calculating the area under a speed vs time plot for this case of constant acceleration. It is just the area of a triangle with the base being the total elapsed time and the height being the final speed.

The elapsed time is ‘t’ and the final speed is given by ‘at’.

Distance traveled = 0.5 base * height

= 0.5 t at

$$x = \frac{at^2}{2}$$

r. Where did the factor of 1/2 come from?

Now we can finally see where the factor of 0.5 came from: it came from the equation for the area of a triangle.

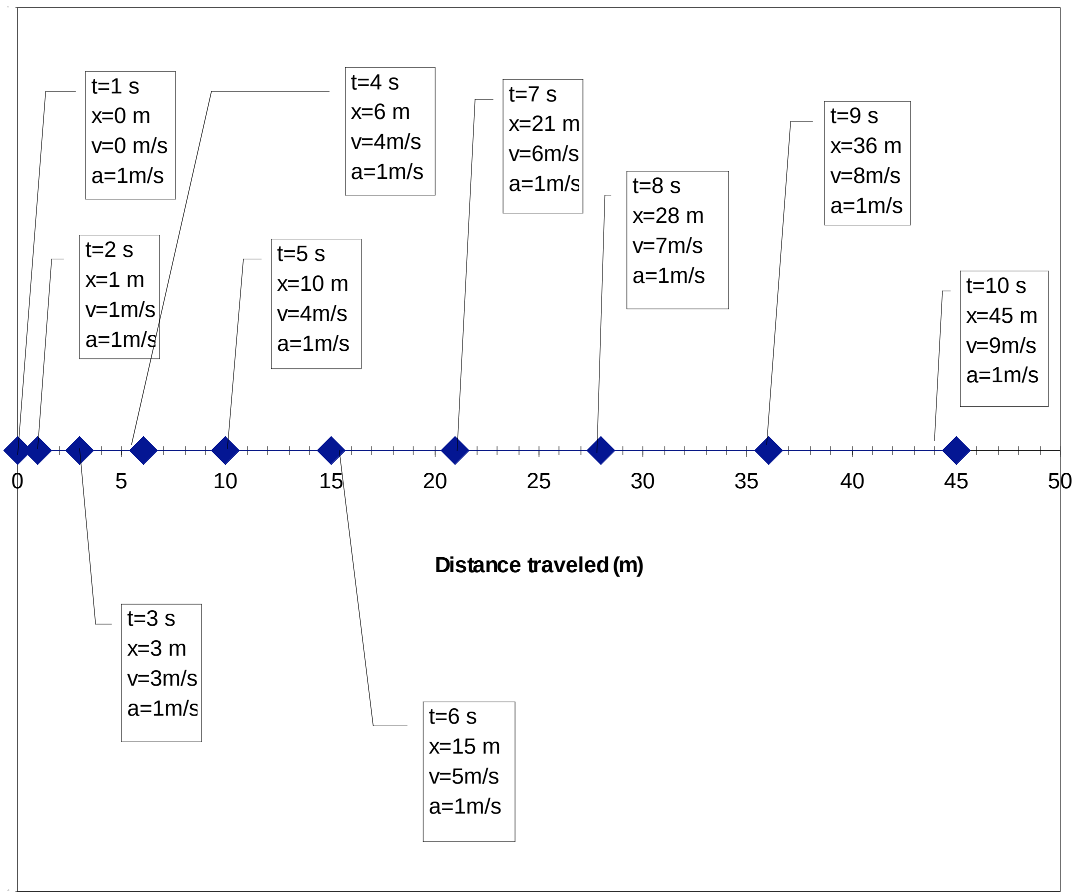
s. Note that since v_f =at, then t= v_f/a, the x-axis (time axis) can be written as v_f/a. The distance, which is the area under the curve of speed (v) vs time (v_f/a) curve, can be written how?

Distance traveled = 0.5 base * height

$$= 0.5 \frac{v_f}{2} * v_f$$

$$x = \frac{v_f^2}{2a}$$

t. On an axis showing the distance traveled in meters, show the time, position, speed, and acceleration after time intervals of 1 s from 0 to 10 s.



23. Suppose you are designing freeway on-ramps for the interstate highway system. How long should the on-ramp be if the speed limit on the freeway is:
a. 55 mph
b. 65 mph
c. 75 mph

Use $v^2 = 2ax$, since the car should accelerate to the speed limit so that it can merge safely. A typical car can accelerate to 60 mi/hr in about 10 s, corresponding to an acceleration of 60mi/hr/10 s = 88ft/s/10s = 8.8 ft/s^2. Let's assume a value of 8 ft/s^2.

a. $x = v^2/2a = \dfrac{(55\text{mi/hr} * 88\text{ft/s}/60\text{mi/hr})^2}{2*(8\text{ft/s}^2)} = 407$ ft.

b. $x = v^2/2a = \dfrac{(65\text{mi/hr} * 88\text{ft/s}/60\text{mi/hr})^2}{2*(8\text{ft/s}^2)} = 568$ ft.

c. $x = v^2/2a = \dfrac{(75\text{mi/hr} * 88\text{ft/s}/60\text{mi/hr})^2}{2*(8\text{ft/s}^2)} = 756$ ft.

According to the California Department of Transportation Manual (http://www.dot.ca.gov/hq/oppd/hdm/chaters/t504.htm)
a typical freeway distance for the acceleration lane is 330 m = 1083 ft.
The values determined by our calculation above are the minimum distance needed to accelerate, so the 1083 feet value includes a reasonable safety margin for a car to safely merge. A large truck accelerates much slower than this so truck drivers must be especially careful when they merge onto a freeway.

24. How do we understand an expression such as (10mi/hr)/s? It will become clearer in section 4. But let's at least analyze it in the standard way we analyze fractions. Recall that dividing by a number is the same as multiplying by 1/number.

So: $(10\text{ mi/hr})/\text{s} = \dfrac{\frac{10\text{mi}}{\text{hr}}}{\text{s}} = \dfrac{10\text{ mi}}{\text{hr}} * \dfrac{1}{\text{s}} = \dfrac{10\text{ mi}}{\text{hr*s}}$

Similarly $(-10\text{m/s})/\text{s} = \dfrac{\frac{-10\text{m}}{\text{s}}}{\text{s}} = \dfrac{-10\text{ m}}{\text{s}} * \dfrac{1}{\text{s}} = \dfrac{-10\text{ m}}{\text{s*s}} = \dfrac{-10\text{ m}}{\text{s}^2}$

25. "How do you measure a year?"
Did you ever think that unit conversions could be the inspiration behind a hit Broadway musical? It's true.
In the hit Broadway musical 'Rent,' there is a song called "Seasons of Love."
The lyrics are shown below

> ALL
> Five hundred twenty-five thousand six hundred minutes
> Five hundred twenty-five thousand moments so dear
> Five hundred twenty-five thousand six hundred minutes
> How do you measure - measure a year?
>
> In daylights, in sunsets, in midnights, in cups of coffee
> In inches, in miles, in laughter, in strife
> In five hundred twenty-five thousand six hundred minutes
> How do you measure a year in the life?
>
> How about love? How about love?
> How about love? Measure in love -
> Seasons of love...
> Seasons of love...
>
> WOMAN
> Five hundred twenty-five thousand six hundred minutes
> Five hundred twenty-five thousand journeys to plan
> Five hundred twenty-five thousand six hundred minutes
> How do you measure the life of a woman or a man?
>
> MAN
> In truths that she learned or in times that he cried
> In bridges he burned or the way that she died
>
> ALL
> It's time now to sing out though the story never ends
> Let's celebrate, remember year in the life of friends
> Remember the love... Remember the love...
> Remember the love... Measure in love -
>
> WOMAN
> Measure, measure your life in love...
>
> ALL
> Seasons of love...
> Seasons of love...

a. Show that there are 525,600 minutes in a year.
b. How many daylights are there in a year?
c. How many sunsets are there in a year?
d. How many midnights are there in a year?
e. How many cups of coffee do you drink in a year?
f. The earth is 93,000,000 miles from the sun. How many inches does the earth travel around the sun in a year?
g. How many miles does the earth travel around the sun in a year?
h. How many times do you laugh in a year?

i. How many times do you have strife in your life in a year?
j. Write some lyrics to a song that includes unit conversions.

Answers:

a. Show that there are 525,600 minutes in a year.

$$\frac{60\ \text{min}}{\cancel{\text{h}}} \times \frac{24\ \cancel{\text{h}}}{\cancel{\text{day}}} \times \frac{365\ \cancel{\text{days}}}{\text{year}} = \frac{525{,}600\ \text{min}}{\text{year}}$$

b. How many daylights are there in a year?

$$\frac{1\ \text{daylight}}{\text{day}} \times \frac{365\ \text{days}}{\text{year}} = \frac{365\ \text{daylights}}{\text{year}}$$

c. How many sunsets are there in a year?

$$\frac{1\ \text{sunset}}{\text{day}} \times \frac{365\ \text{days}}{\text{year}} = \frac{365\ \text{sunsets}}{\text{year}}$$

d. How many midnights are there in a year?

$$\frac{1\ \text{midnight}}{\text{day}} \times \frac{365\ \text{days}}{\text{year}} = \frac{365\ \text{midnights}}{\text{year}}$$

e. How many cups of coffee do you drink in a year?

$$\frac{2\ \text{cups}}{\text{day}} \times \frac{365\ \text{days}}{\text{year}} = \frac{730\ \text{cups}}{\text{year}}$$

f. The earth is 93,000,000 miles from the sun. How many inches does the earth travel around the sun in a year?

The earth's orbit is nearly circular, the distance traveled is the circumference of the orbit, and the circumference of a circle is 2 x pi x radius, so:

$$\text{Distance} = 2 \times 3.14 \times 93 \times 10^6\ \text{miles} \times \frac{5280\ \text{ft}}{\text{mile}} \times \frac{12\ \text{in}}{\text{ft}} = 3.70 \times 10^{13}\ \text{inches}$$

g. How many miles does the earth travel around the sun in a year?

The earth's orbit is nearly circular, ,the distance traveled is the circumference of the orbit, and the circumference of a circle is 2 x pi x radius, so:

$$\text{Distance} = 2 \times 3.14 \times 93 \times 10^6\ \text{miles} = 5.84 \times 10^8\ \text{miles}$$

h. How many times do you laugh in a year?

$$\frac{20 \text{ laughs}}{\text{day}} \times \frac{365 \text{ days}}{\text{year}} = \frac{7300 \text{ laughs}}{\text{year}}$$

i. How many times do you have strife in your life in a year?

$$\frac{1 \text{ strife}}{\text{week}} \times \frac{52 \text{ weeks}}{\text{year}} = \frac{52 \text{ strifes}}{\text{year}}$$

j. Write some lyrics to a song that includes unit conversions.

Good luck and have fun. Perform it in front of the class for extra enjoyment!

26. Hydroplaning can occur when driving on roads on which about 0.2 inch of water has accumulated. Suppose there is a heavy downpour where it is raining at a rate of 4 inches per hour and there is depression on the road that does not drain. All of the water in this depression does however get splashed away when a car drives over it. (This problem is based on a crash that happened to a friend of mine.)

a. How long will it take for the depression to fill to the point where it will lead to hydroplaning?

$$\text{Time} = \frac{0.20 \text{ in}}{4 \text{ in/hr}} = 0.05 \text{ hr} \times \frac{60\text{min}}{1 \text{ hr}} = 3 \text{ min}$$

b. Suppose that you are driving in the car pool (diamond) lane and a car was in that lane 2 minutes ago. Will you hydroplane and probably lose control of your car?

No. The water in the depression will have accumulated to a depth of less than 0.2 inches so your car will not hydroplane.

c. Suppose that you are driving in the car pool (diamond) lane and no other car has used that lane for the last 4 minutes. Will you hydroplane and probably lose control of your car?

Yes. The water in the depression will have accumulated to a depth of greater than 0.2 inches so your car will hydroplane.

d. How will the above calculations affect how you drive in the rain?

If there are not many cars driving in my lane, I should slow down a lot because it is likely that I will hydroplane in areas of the road where there are small depressions. It is best if I drive in the lanes that are most used, not ones that are hardly used at all.

Simulation Tool Development

1.

a. Develop a scale model car that shows the distance traveled in 1 s at speeds from 10 to 70 mph in increments of 10 mph. Use a scale of 1 in = 100 ft. Include a marker on each scale model car that shows the 160 ft distance that is illuminated by a car's headlights.

b. Develop a scale model car that shows the distance traveled in 1 s at speeds from 10 to 70 m/s in increments of 10 m/s. Use a scale of 1 cm = 10 m.

2. The stopping distance for a car is equal to the distance it travels during your reaction time (reaction distance) plus the distance it travels while the brakes are applied (braking distance). The typical deceleration of a car is 17 ft/s^2 and a typical reaction time is 1.5 s. Recall the equation for calculating stopping distance is $x = v^2/2a$, where v is the initial speed of the car, and the equation for calculating the distance traveled at constant speed is x = vt. Calculate the reaction distance, braking distance, and stopping distance for a car traveling at the following speeds.

a. 10 mph
b. 20 mph
c. 30 mph
d. 40 mph
e. 50 mph
f. 60 mph
g. 70 mph
h. Make a table summarizing the above data.
i. Make a plot summarizing the above data.
j. Calculate the braking time for the car for each of the cases a-g. Recall that for the case of constant acceleration, $t = \frac{v}{a}$,

where v is the initial speed and the deceleration is a. Generate a table and a graph for the reaction time, braking time, and total stopping time for each speed.

k. Generate a scale model car and map (or use car D and map A) to simulate these situations.

The stopping distance for a car is equal to the distance it travels during your reaction time (reaction distance) plus the distance it travels while the brakes are applied (braking distance). The typical deceleration (a) of a car is 17 ft/s^2 and a typical reaction time (t) is 1.5 s. Calculate the reaction distance (vt), braking distance ($v^2/(2a)$), and total stopping distance ($vt + v^2/(2a)$) for a car traveling at the following speeds (v).

(a) 10 mph

Reaction distance = speed x reaction time (recall that speed is a constant during the reaction time)

$$\text{Reaction distance} = \frac{10\text{ mi}}{\text{h}} \times 1.5\text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60\text{ mi}} = 22\text{ ft}$$

{Recall that $\frac{88\text{ ft}}{\text{s}} = \frac{60\text{ mi}}{\text{h}}$ so $\frac{88\text{ ft}}{\text{s}} \times \frac{\text{h}}{60\text{ mi}} = 1$

This is a simple and useful conversion to remember. Or the student could use the previously found speed in ft/s as is done in the braking distance calculation below}

Braking distance = $\frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$ (During this time, the deceleration is constant)

$$= \frac{14.7^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 6.4 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 22 ft + 6.4 ft = 28.4 ft.

(b) 20 mph

Reaction distance $= \frac{20 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 44 \text{ ft}$

Braking distance = $\frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$

$$= \frac{29.3^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 25.2 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 44 ft + 25.2 ft = 69.2 ft.

(c) 30 mph

Reaction distance $= \frac{30 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 66 \text{ ft}$

Braking distance = $\frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$

$$= \frac{44^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 56.9 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 66 ft + 56.9 ft = 122.9 ft.

(d) 40 mph

$$\text{Reaction distance} = \frac{40 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 88 \text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{58.7^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 101.3 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 88 ft + 101.3 ft = 189.3 ft.

(e) 50 mph

$$\text{Reaction distance} = \frac{50 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 110 \text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{73.3^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 158 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 110 ft + 158 ft = 268 ft.

(f) 60 mph

$$\text{Reaction distance} = \frac{60 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 132 \text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{88^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 228 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 132 ft + 228 ft = 360 ft.

(g) 70 mph

$$\text{Reaction distance} = \frac{70\text{ mi}}{\text{h}} \times 1.5\text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60\text{ mi}} = 154\text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

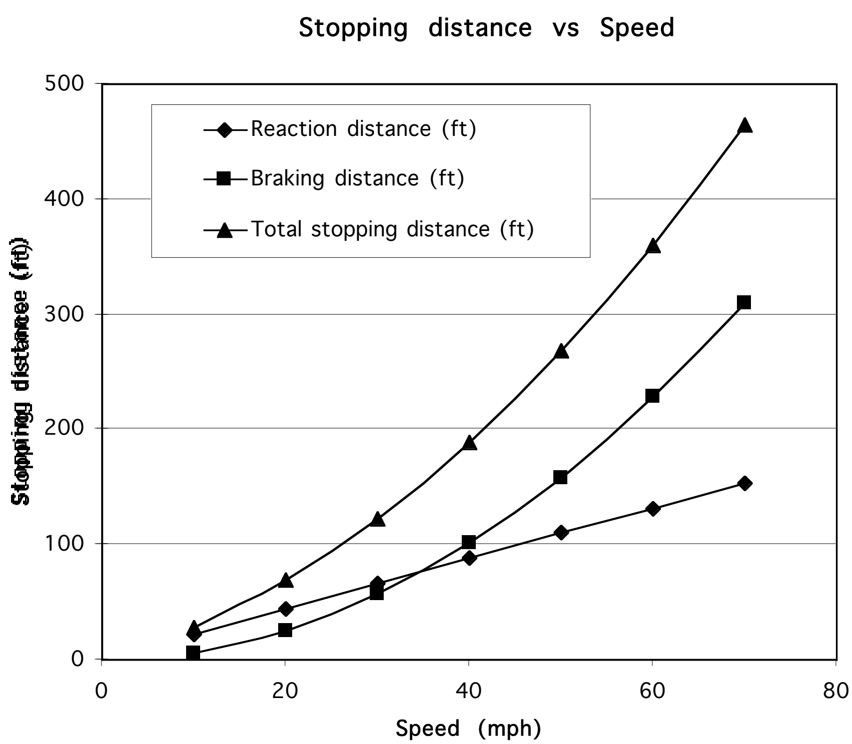
$$= \frac{102.7^2\text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 310\text{ ft}$$

So stopping distance = reaction distance + braking distance = 154 ft + 310 ft = 464 ft.

(h)

Speed (mph)	Reaction distance (ft)	Braking distance (ft)	Total stopping distance (ft)
10	22	6.4	28.4
20	44	25.2	69.2
30	66	56.9	122.9
40	88	101.3	189.3
50	110	158	268
60	132	228	360
70	154	310	464

(i)



i.

10 mph = $14.7 \frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{14.7\ ft}{s} \frac{s^2}{17\ ft} = 0.9\ s$

20 mph = $29.3 \frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{29.3\ ft}{s} \frac{s^2}{17\ ft} = 1.7\ s$

30 mph = $44 \frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{44\ ft}{s} \frac{s^2}{17\ ft} = 2.6\ s$

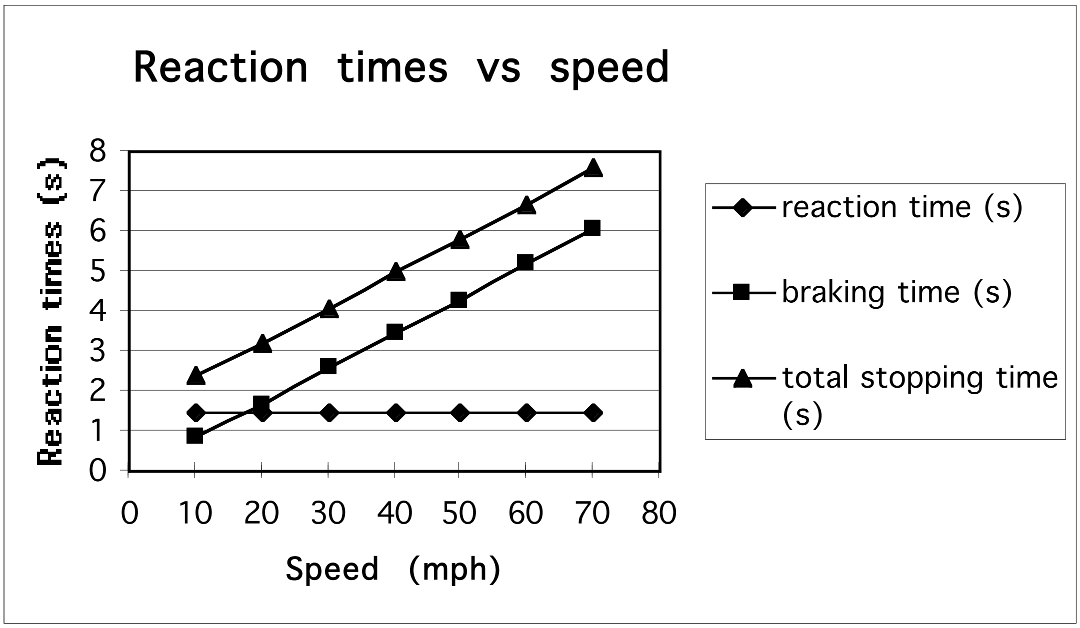
40 mph = $58.7 \frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{58.7\ ft}{s} \frac{s^2}{17\ ft} = 3.5\ s$

50 mph = $73.3 \frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{73.3\ ft}{s} \frac{s^2}{17\ ft} = 4.3\ s$

60 mph = $88 \frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{88\ ft}{s} \frac{s^2}{17\ ft} = 5.2\ s$

70 mph = $103 \frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{103\ ft}{s} \frac{s^2}{17\ ft} = 6.1\ s$

Speed (mph)	Reaction time (s)	Braking time (s)	Total stopping time (s)
10	1.5	.9	2.4
20	1.5	1.7	3.2
30	1.5	2.6	4.1
40	1.5	3.5	5.0
50	1.5	4.3	5.8
60	1.5	5.2	6.7
70	1.5	6.1	7.6



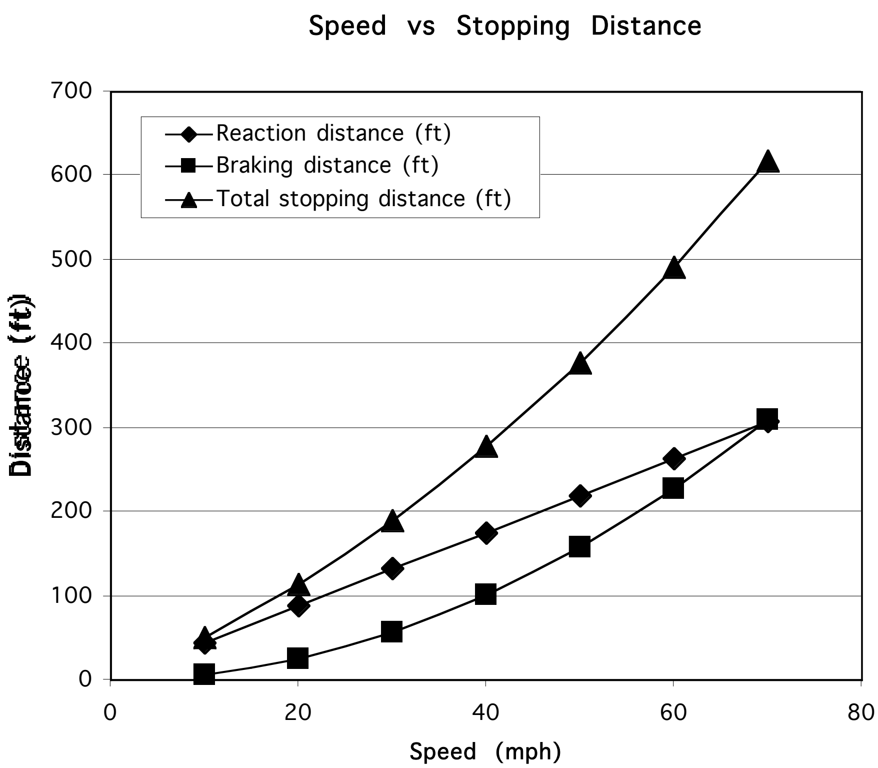
Develop a scale model car that shows the reaction distance, braking distance, and total stopping distance for speeds for 10 to 70 mph in increments of 10 mph. Include the reaction time, braking time, and total stopping time in the scale model car simulation tool. Also include a marker that shows the 160 ft distance that is illuminated by a car's headlights. Use the scale of 1 in = 100 ft.

3. Redo the calculations of problem 2 assuming that you are drunk and your reaction time is twice as long (3 s instead of 1.5 s).

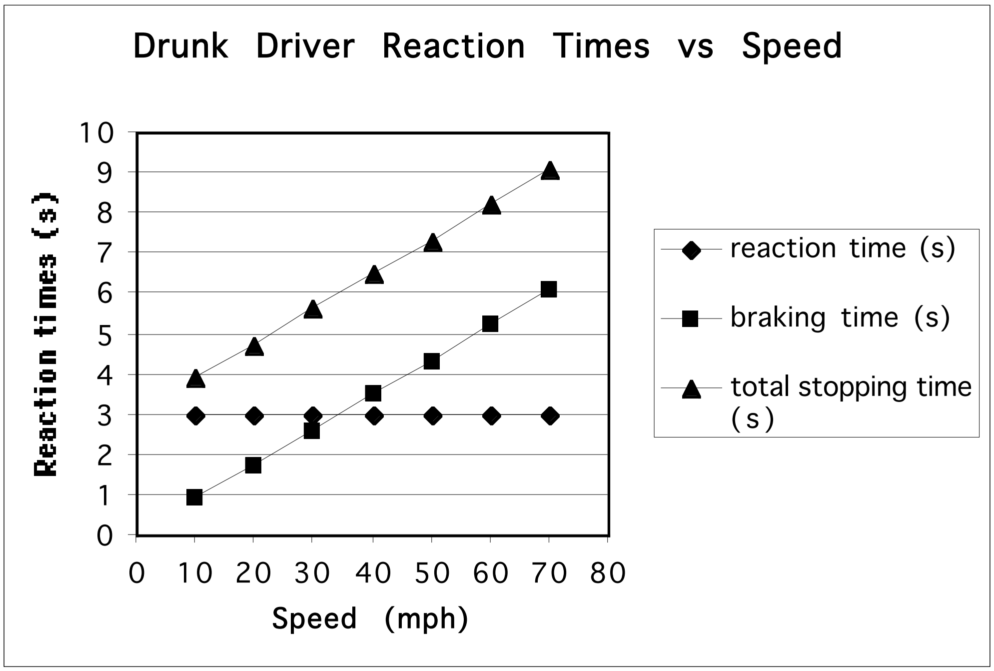
Generate a drunk driver scale model and map (or use car E and map A) to simulate these situations.

The reaction distance will be double the distances calculated in problem 5 because the reaction distance is directly proportional to the reaction time. The braking distance will remain the same because we are assuming it is not affected.

Speed (mph)	Reaction distance (ft)	Braking distance (ft)	Total stopping distance (ft)
10	44	6.4	50.4
20	88	25.2	113.2
30	132	56.9	188.9
40	176	101.3	277.3
50	220	158	378
60	264	228	492
70	308	310	618



Speed (mph)	Reaction time (s)	Braking time (s)	Total stopping time (s)
10	3.0	0.9	3.9
20	3.0	1.7	4.7
30	3.0	2.6	5.6
40	3.0	3.5	6.5
50	3.0	4.3	7.3
60	3.0	5.2	8.2
70	3.0	6.1	9.1



Develop a scale model car that shows the reaction distance, braking distance, and total stopping distance for speeds for 10 to 70 mph in increments of 10 mph. Include the reaction time, braking time, and total stopping time in the scale model car simulation tool. Also include a marker that shows the 160 ft distance that is illuminated by a car's headlights. Use the scale of 1 in = 100 ft.

4. Redo the calculations of problem 2 assuming that the road is wet. The deceleration of a car on a wet road is apparently 75-95% of the dry road value (see Appendix A) - assuming that there is no hydroplaning. So assume that the wet road deceleration is 13 ft/s^2 instead of the 17ft/s^2 that we have using for a dry road.

(a) 10 mph

Reaction distance = speed x reaction time (recall that speed is a constant during the reaction time)

$$\text{Reaction distance} = \frac{10\text{ mi}}{\text{h}} \times 1.5\text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60\text{ mi}} = 22\text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{14.7^2\text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 13\text{ft}} = 8.3\text{ ft}$$

So stopping distance = reaction distance + braking distance = 22 ft + 8.3 ft = 20.3 ft.

(b) 20 mph

$$\text{Reaction distance} = \frac{20\text{ mi}}{\text{h}} \times 1.5\text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60\text{ mi}} = 44\text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{29.3^2\text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 13\text{ft}} = 33.0\text{ ft}$$

So stopping distance = reaction distance + braking distance = 44 ft + 33.0 ft = 77.0 ft.

(c) 30 mph

$$\text{Reaction distance} = \frac{30\text{ mi}}{\text{h}} \times 1.5\text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60\text{ mi}} = 66\text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{44^2\text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 13\text{ft}} = 74.5\text{ ft}$$

So stopping distance = reaction distance + braking distance = 66 ft + 74.5 ft = 140.5 ft.

(d) 40 mph

$$\text{Reaction distance} = \frac{40 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 88 \text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{58.7^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 13\text{ft}} = 132.5 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 88 ft + 132.5 ft = 220.5 ft.

(e) 50 mph

$$\text{Reaction distance} = \frac{50 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 110 \text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{73.3^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 13\text{ft}} = 206.6 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 110 ft + 206.6 ft = 316.6 ft.

(f) 60 mph

$$\text{Reaction distance} = \frac{60 \text{ mi}}{\text{h}} \times 1.5 \text{ s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60 \text{ mi}} = 132 \text{ ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{88^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 13\text{ft}} = 297.8 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 132 ft + 297.8 ft = 429.8 ft.

(g) 70 mph

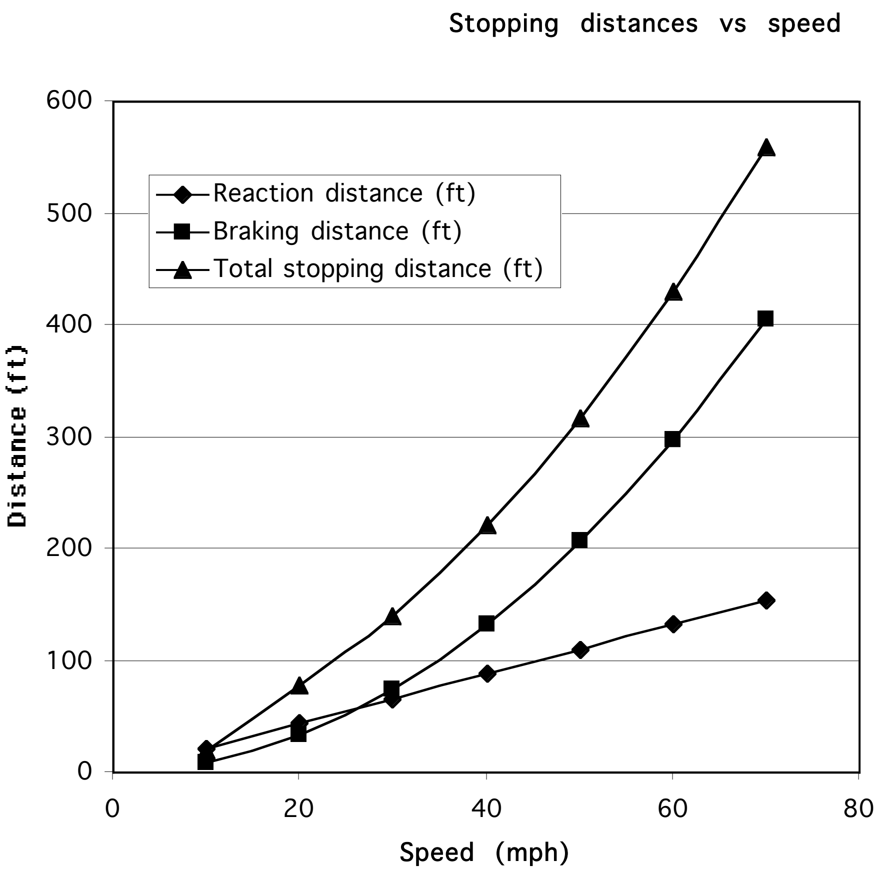
$$\text{Reaction distance} = \frac{70\ \text{mi}}{\text{h}} \times 1.5\ \text{s} \times \frac{88\text{ft}}{\text{s}} \times \frac{\text{h}}{60\ \text{mi}} = 154\ \text{ft}$$

$$\text{Braking distance} = \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$$

$$= \frac{102.7^2\ \text{ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 13\text{ft}} = 405.7\ \text{ft}$$

So stopping distance = reaction distance + braking distance = 154 ft + 405.7 ft = 559.7 ft.

Speed (mph)	Reaction distance (ft)	Braking distance (ft)	Total stopping distance (ft)
10	22	8.3	30.3
20	44	33	77
30	66	74.5	140.5
40	88	132.5	220.5
50	110	206.6	316.6
60	132	297.8	429.8
70	154	405.7	559.7



10 mph = 14.7 $\frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{14.7\ ft}{s} \frac{s^2}{13\ ft} = 1.1\ s$

20 mph = 29.3 $\frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{29.3\ ft}{s} \frac{s^2}{13\ ft} = 2.3\ s$

30 mph = 44 $\frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{44\ ft}{s} \frac{s^2}{13\ ft} = 3.4\ s$

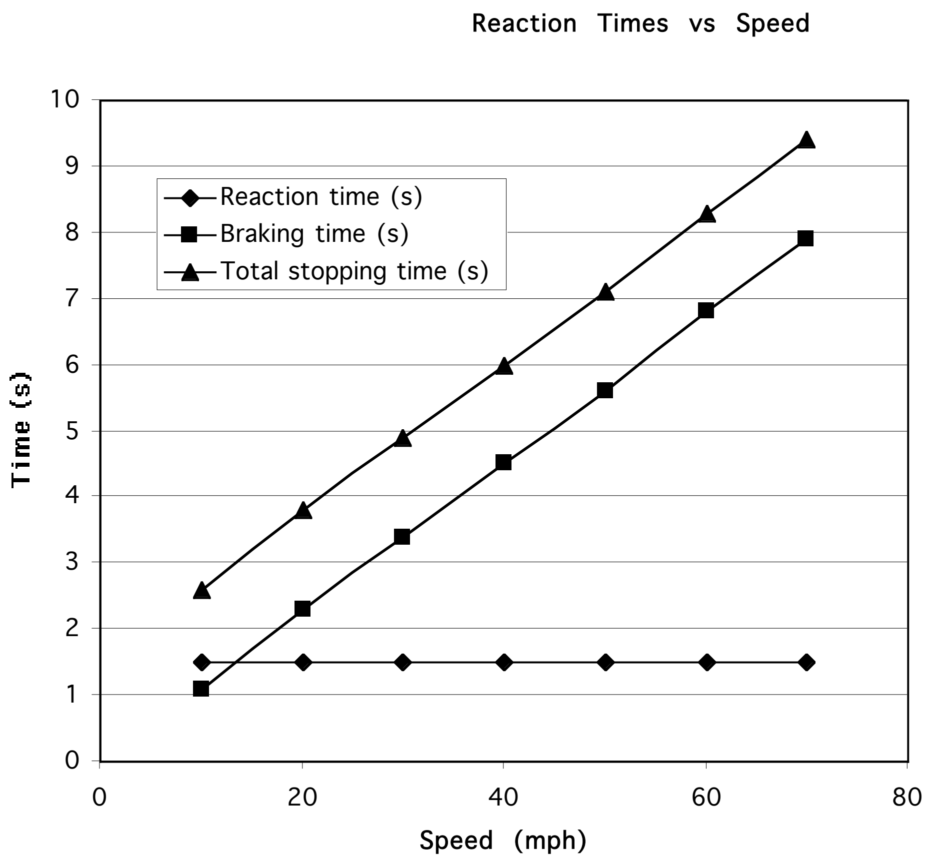
40 mph = 58.7 $\frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{58.7\ ft}{s} \frac{s^2}{13\ ft} = 4.5\ s$

50 mph = 73.3 $\frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{73.3\ ft}{s} \frac{s^2}{13\ ft} = 5.6\ s$

60 mph = 88 $\frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{88\ ft}{s} \frac{s^2}{13\ ft} = 6.8\ s$

70 mph = 103 $\frac{ft}{s}$ so braking time $t = \frac{v}{a} = \frac{103\ ft}{s} \frac{s^2}{13\ ft} = 7.9\ s$

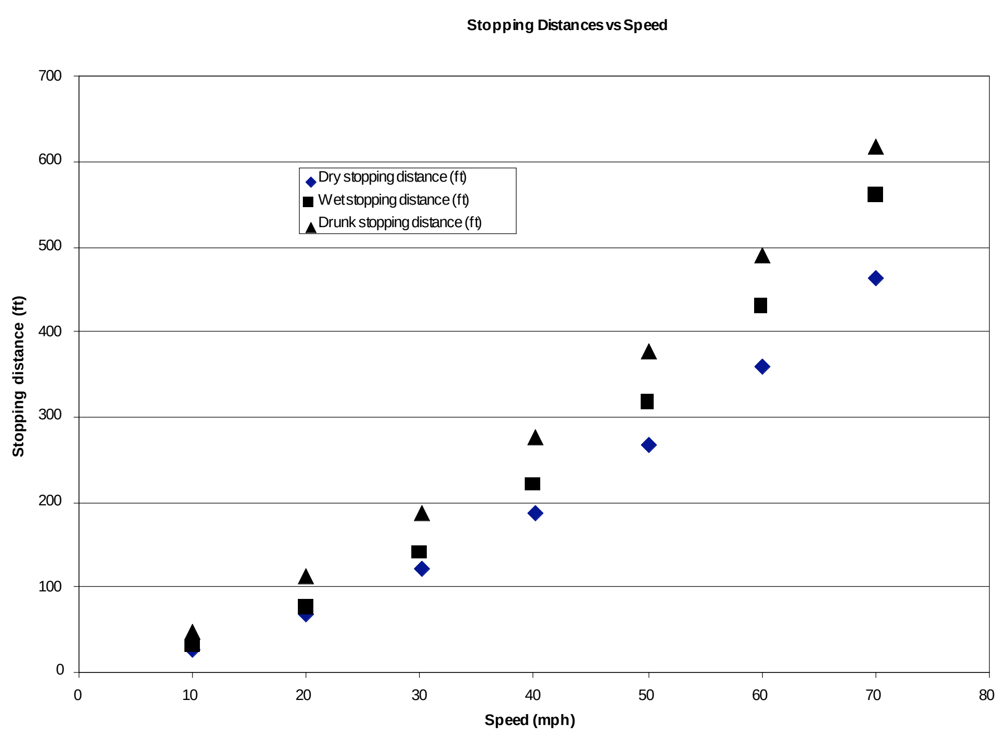
Speed (mph)	Reaction time (s)	Braking time (s)	Total stopping time (s)
10	1.5	1.1	2.6
20	1.5	2.3	3.8
30	1.5	3.4	4.9
40	1.5	4.5	6.0
50	1.5	5.6	7.1
60	1.5	6.8	8.3
70	1.5	7.9	9.4


Reaction Times vs Speed
Time (s)
Speed (mph)
Reaction time (s)
Braking time (s)
Total stopping time (s)
0
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80

5. Compare the total stopping distance and total reaction time for a driver on a dry road, a driver on a wet road and a drunk driver on a dry road. Use the results of the previous problems. Show the result in a table for speeds from 10 to 70 mph in increments of 10 mph. Plot the different stopping distances vs speed. What are the implications of these tabulated results for safe driving?

Speed (mph)	Dry Stopping Distance (ft)	Dry Stopping Time (s)	Wet Stopping Distance (ft)	Wet Stopping Time (s)	Drunk Stopping Distance (ft)	Drunk Stopping Time (s)
10	28	2.4	20	2.6	50	3.9
20	69	3.2	77	3.8	113	4.7
30	123	4.1	141	4.9	189	5.6
40	189	5.0	221	6.0	277	6.5
50	268	5.8	317	7.1	378	7.3
60	360	6.7	430	8.3	492	8.2
70	464	7.6	560	9.4	618	9.1

The stopping distances increase dramatically as the speed increases. The stopping distances for driving on wet roads and for drunk drivers are much greater than that for dry roads and sober drivers. The implications are: slow down when the roads are wet (drive about 10 mph slower than your typical speed) because the stopping distances are greater and don't drive if drunk.



Investigation # 3 – Distance and Speed

1. Using the slots in a sidewalk (or tiles in the floor or fence posts or markers on a field or parking lot spaces or light poles or any other regularly spaced marker), walk for 20 s at a steady pace. Have a second student note the number of sidewalk slots passed by the first student at the end of
   a. 0 s
   b. 5 s
   c. 10 s
   d. 15 s
   e. 20 s
2. Make a table of time and slot number passed at that time.
3. Measure the length between slots in meters.
4. Add a third column to the table: total distance traveled.
5. Add a fourth column to the table: distance traveled during the preceding 5 s.
6. Make a plot of total distance traveled vs. time – plot the 4 data points.

Your average speed is how fast you've walked and can be calculated by dividing the total distance traveled by the time it takes to walk that total distance.

7. Add a fifth column to your table: your speed.
8. Calculate your average speed.
9. Plot your average speed vs. time after 5, 10, and 15 s.
10. Calculate your speed during a given time interval by determining how far you walked during that time period and dividing that distance by the time interval.
11. Plot your speed during the interval vs. the time in the middle of the interval.
12. Repeat parts 1-10 except stop between 5 and 10 s.
13. How did the average speed and the speed during each time interval compare for these two experiments. Why did they differ?
14. Using your typical average speed, calculate how long it will take you to walk:
    a. 100 m
    b. 1 km
    c. 1 mile
    d. 5 miles.

Investigation # 4: Using a Map

1. Obtain a map (or draw a map using your best estimates - make sure your map includes a distance scale) that includes your present location and the location of a nearby place that you have visited or would like to visit.

   (a) Using the map scale, calculate the distance of a route that will take you from your present location to your destination. Your trip must include at least 3 turns, and travel at least 2 different speeds.

   (b) Calculate how long it will take you to travel to your destination.

   (For a residential street, assume that you will travel at 25 mph, for a non-residential road, assume that you will travel at 35 miles/hr; for major freeways and highways, assume that you will travel at 60 miles/hr. If you plan on walking, assume that you walk at 3 miles/hr. If you run, assume that you can cover 6 miles in 1 hour.)

2. Same as problem 1 above, except that the location that you will visit must be in another state, at least 1000 miles away. Also assume that you will stop for 1 hour for your meals, 0.5 hours for a rest stop, and 10 hours for sleeping, and that you will only travel between 8 AM and 8 PM.

4A Distance: Area Under a Curve

1. What is the area of a rectangle?

   A= length x height

2. a. What is the area of a rectangle with a length of 1 s and a height of 88ft/s?

   A = (1s) * (88ft/s) = 88ft

   b. What is the distance traveled by a car in 1 s at a speed of 88ft/s?

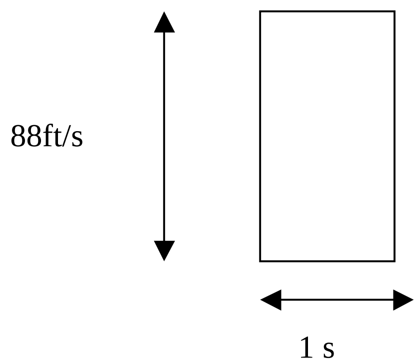
   distance = (v) (t) = (88ft/s)(1 s) = 88ft.

   c. Do you notice any correlation between the distance traveled and the area of a speed vs time rectangle.

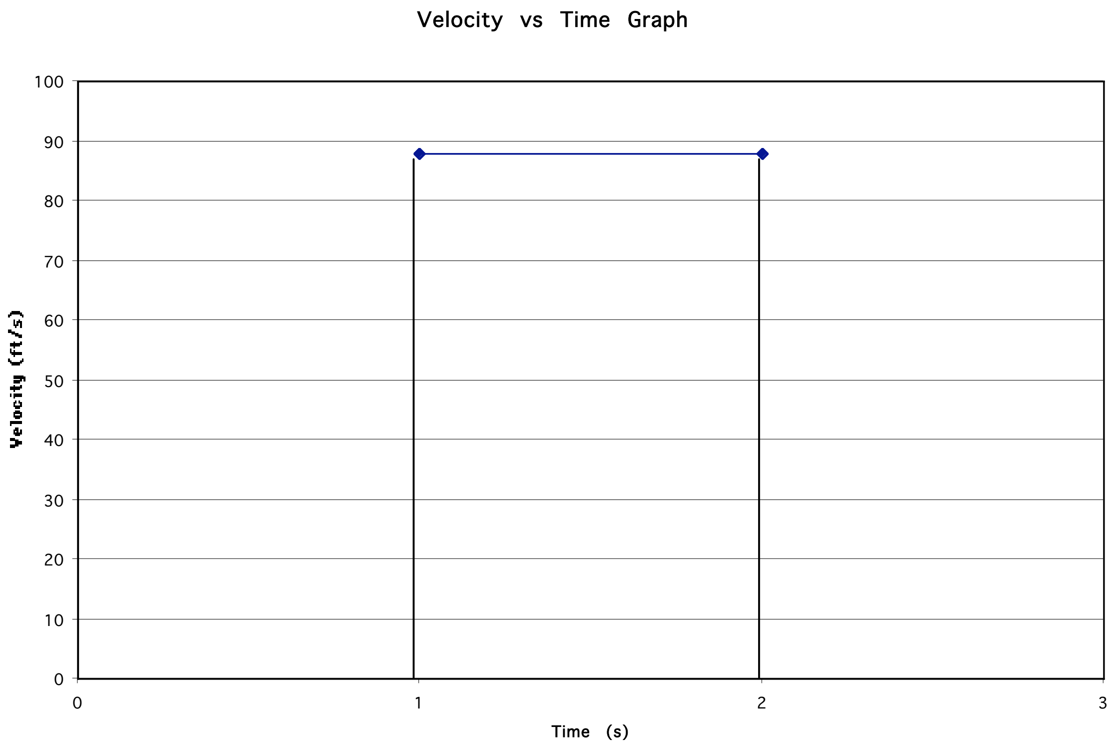
   Yes. the distance traveled is the area of a speed vs time rectangle

   d. Draw this rectangle with time on the horizontal axis and use your drawing to restate the conclusion reached in part c.

   The area of the rectangle below, with time on the horizontal axis and velocity on the vertical axis, indicates the distance traveled.



e. Show the rectangle in part d as a rectangle on a graph where time is the x-axis and velocity is the y-axis.



3. a. What is the area of a rectangle with a length of 1 hr and a height of 60mi/hr?
A = (1hr) * (60mi/hr) = 60 mi.

b. What is the distance traveled by a car in 1 hour at a speed of 60mi/hr?
distance = (v) (t) = (60mi/hr) (1 hr) = 60 mi.

c. Do you notice any correlation between the distance traveled and the area of a speed vs time rectangle.
Yes. the distance traveled is the area of a speed vs time rectangle

d. Draw this rectangle with time on the horizontal axis and use your drawing to restate the conclusion reached in part c.

4B Definitions

1.

Symbol	Sign can be + or -	Always positive
t	time	
delta t or $(t_f - t_i)$	change in time	elapsed time
x	position	
delta x or $(x_f - x_i)$	change in position	distance
$\frac{\text{delta x}}{\text{delta t}}$ or $\frac{(x_f - x_i)}{(t_f - t_i)}$	velocity	speed
$\frac{\text{delta v}}{\text{delta t}}$ or $\frac{(v_f - v_i)}{(t_f - t_i)}$	acceleration	magnitude of acceleration

2. Assume that the line below shows the position of an object in units of 1 m. Let the position of the object be called x. Determine the following:

a. Position of the object at A.

$x = -11m$

b. Position of the object at B.

$x = -10m$

c. Position of the object at C.

$x = -1m$

d. Position of the object at D.

$x = 0m$

e. Position of the object at E.

$x = 1m$

f. Position of the object at F.

$x = 10m$

g. Position of the object at G.

$x = 11m$

h. Distance between A and B. The change in position for object that moves from A to B.

$D = 1m$; $x_f - x_i = -10m - (-11m) = 1m$

i. Distance between C and E. The change in position for object that moves from C to E.

$D = 2m$; $x_f - x_i = 1m - (-1m) = 2m$

j. Distance between B and F. The change in position for object that moves from B to F.

$D = 20m$; $x_f - x_i = 10m - (-10m) = 20m$

k. Distance between G and B. The change in position for object that moves from G to B.

$D = 21m$; $x_f - x_i = -10m - (11m) = -21m$

l. Distance between B and A. The change in position for object that moves from B to A.

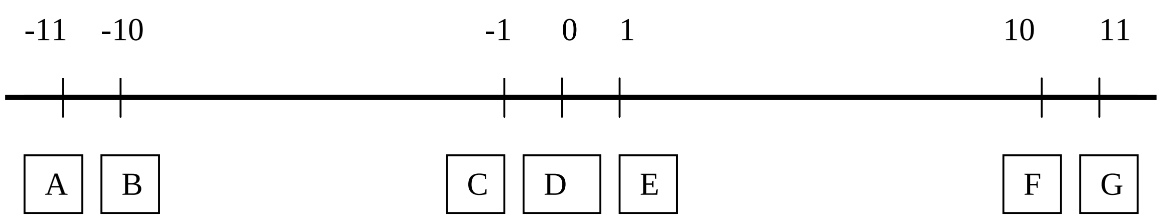
$D = 1m$; $x_f - x_i = -11m - (-10m) = -1m$

m. Distance between A and D. The change in position for object that moves from D to A.

$D = 11m$; $x_f - x_i = -11m - (0m) = -11m$

n. Distance between G and D. The change in position for object that moves from G to D.

$D = 11m$; $x_f - x_i = 0m - (11m) = -11m$



3. Assume that the line in problem 2 shows the velocity of an object in units of 1 m/s. Let the velocity of the object be called v. Determine the following:

a. Velocity and speed of the object at A.

v = -11m/s; speed = 11m/s

b. Velocity and speed of the object at B.

v = -10m/s; speed = 10m/s

c. Velocity and speed of the object at C.

v = -1m/s; speed = 1m/s

d. Velocity and speed of the object at D.

v = 0m/s; speed = 0m/s

e. Velocity and speed of the object at E.

v = 1m/s; speed = 1m/s

f. Velocity and speed of the object at F.

v = 10m/s; speed = 10m/s

g. Velocity and speed of the object at G.

v = 11m/s; speed = 11m/s

h. Change in speed for an object that moves from A and B. The change in velocity for object that moves from A to B.

change in speed = 1m/s; change in velocity = $v_f - v_i$ = 1m/s

i. Change in speed for an object that moves from C and E. The change in velocity for object that moves from C to E.

change in speed = 2m/s; change in velocity = $v_f - v_i$ = 2m/s

j. Change in speed for an object that moves from B and F. The change in velocity for object that moves from B to F.

change in speed = 20m/s; change in velocity = $v_f - v_i$ = 20m/s

k. Change in speed for an object that moves from G and B. The change in velocity for object that moves from G to B.

change in speed = 21m/s; change in velocity = $v_f - v_i$ = -21m/s

l. Change in speed for an object that moves from B and A. The change in velocity for object that moves from B to A.

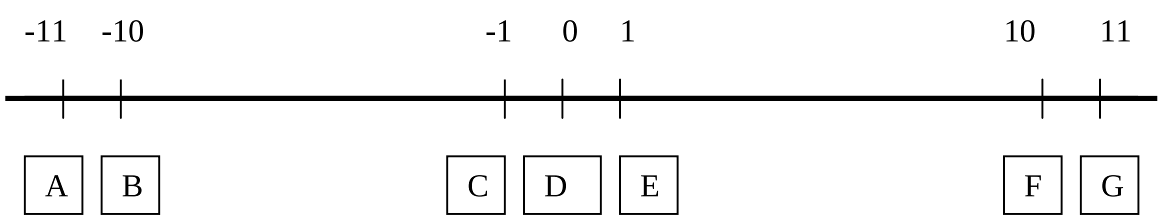
change in speed = 1m/s; change in velocity = $v_f - v_i$ = -1m/s

m. Change in speed for an object that moves from A and D. The change in velocity for object that moves from D to A.

change in speed = 11m/s; change in velocity = $v_f - v_i$ = -11m/s

n. Change in speed for an object that moves from G and D. The change in velocity for object that moves from G to D.

change in speed = 11m/s; change in velocity = $v_f - v_i$ = -11m/s



4. Assume that the line in problem 2 shows the time of an object in units of 1 s. Let the time of the object be called t. Determine the following:

a. Time of the object at A.

t = -11s

b. Time of the object at B.

t = -10s

c. Time of the object at C.

t = -1s

d. Time of the object at D.

t = 0s

e. Time of the object at E.

t = 1s

f. Time of the object at F.

t = 10s

g. Time of the object at G.

t = 11s

h. Elapsed time between A and B. The change in time for object that moves from A to B.

elapsed time = 1s; change in time = $t_f - t_i$ = 1s

i. Elapsed time between C and E. The change in time for object that moves from C to E.

elapsed time =2s; change in time = $t_f - t_i$ = 2s

j. Elapsed time between B and F. The change in time for object that moves from B to F.

elapsed time = 20s; change in time = $t_f - t_i$ = 20s

k. Elapsed time between G and B. The change in time for object that moves from G to B,

elapsed time = 21s; change in time = $t_f - t_i$ = -21s

l. Elapsed time between B and A. The change in time for object that moves from B to A.

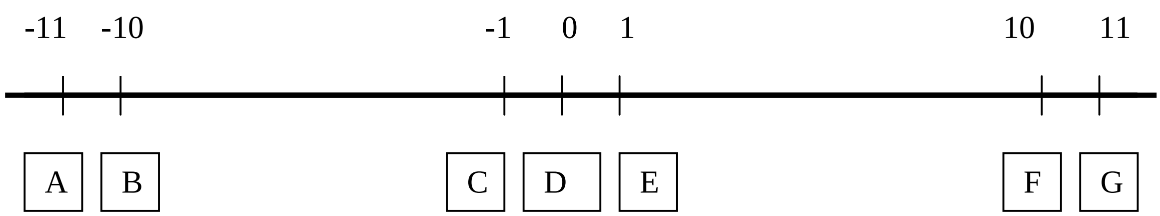
elapsed time = 1s; change in time = $t_f - t_i$ = -1s

m. Elapsed time between A and D. The change in time for object that moves from D to A.

elapsed time = 11; change in time = $t_f - t_i$ = -11s

n. Elapsed time between G and D. The change in time for object that moves from G to D.

elapsed time = 11s; change in time = $t_f - t_i$ = -111s



5. Assume that the line in problem 2 shows the acceleration of an object in units of 1 m/s^2. Let the acceleration of the object be called a. Determine the following:

a. Acceleration and magnitude of the acceleration of the object at A.

a = -11m/s^2; magnitude of acceleration = 11m/s^2

b. Acceleration and magnitude of the acceleration of the object at B.

a = -10m/s^2; magnitude of acceleration = 10m/s^2

c. Acceleration and magnitude of the acceleration of the object at C.

a = -1m/s^2; magnitude of acceleration = 1m/s^2

d. Acceleration and magnitude of the acceleration of the object at D.

a = 0m/s^2; magnitude of acceleration = 0m/s^2

e. Acceleration and magnitude of the acceleration of the object at E.

a = 1m/s^2; magnitude of acceleration = 1m/s^2

f. Acceleration and magnitude of the acceleration of the object at F.

a = 10m/s^2; magnitude of acceleration = 10m/s^2

g. Acceleration and magnitude of the acceleration of the object at G.

a = 11m/s^2; magnitude of acceleration = 11m/s^2

h. Change in magnitude of the acceleration for an object that moves from A and B. The change in acceleration for object that moves from A to B.

change in magnitude of acceleration = 1m/s^2; change in a = $a_f - a_i$ = 1m/s^2

i. Change in magnitude of the acceleration for an object that moves from C and E. The change in acceleration for object that moves from C to E.

change in magnitude of acceleration = 2m/s^2; change in a = $a_f - a_i$ = 2m/s^2

j. Change in magnitude of the acceleration for an object that moves from B and F. The change in acceleration for object that moves from B to F.

change in magnitude of acceleration = 20m/s^2; change in a = $a_f - a_i$ =20m/s^2

k. Change in magnitude of the acceleration for an object that moves from G and B. The change in acceleration for object that moves from G to B.

change in magnitude of acceleration = 21m/s^2; change in a = $a_f - a_i$ = -21m/s^2

l. Change in magnitude of the acceleration for an object that moves from B and A. The change in acceleration for object that moves from B to A.

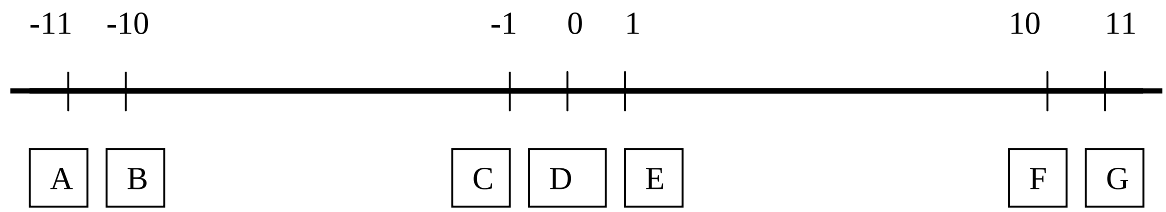
change in magnitude of acceleration = 1m/s^2; change in a = $a_f - a_i$ = -1m/s^2

m. Change in magnitude of the acceleration for an object that moves from A and D. The change in acceleration for object that moves from D to A.

change in magnitude of acceleration = 11m/s^2; change in a = $a_f - a_i$ = -11m/s^2

n. Change in magnitude of the acceleration for an object that moves from G and D. The change in acceleration for object that moves from G to D.

change in magnitude of acceleration = 11m/s^2; change in a = $a_f - a_i$ = -11m/s^2



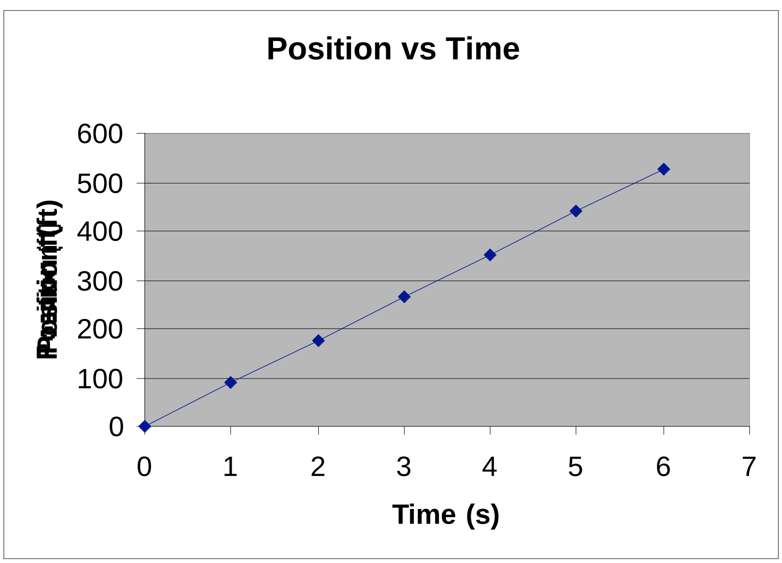
Investigation #5 – Position, speed, and acceleration

1. A car is initially at a position of x = 0 ft. It is traveling in the positive direction at a speed of 88 ft/s. What is the car's position after:

a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. Draw a graphical representation of these results.
h. Describe the motion of the car.

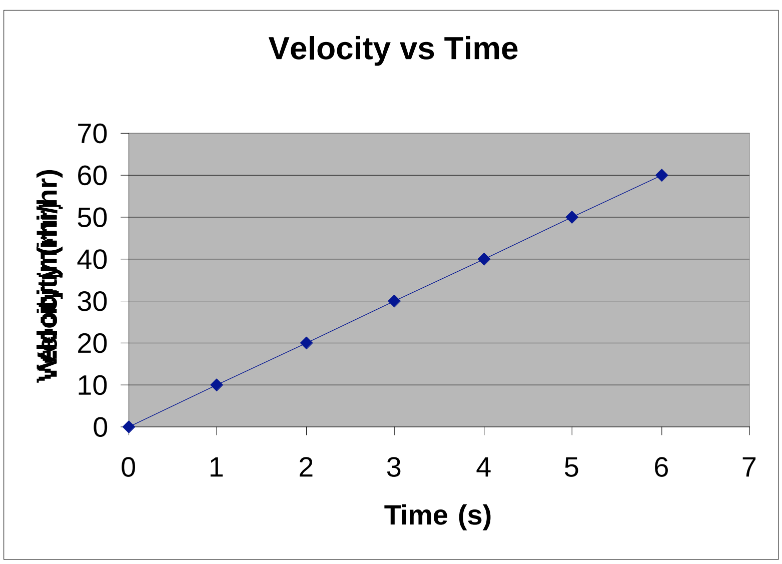
Solution:

1. $x_f = x_i + vt$
a. x = 0 ft + vt = (88 ft/s)(1 s) = 88 ft
b. x = 0 ft + vt = (88 ft/s)(2 s) = 176 ft
c. x = 0 ft + vt = (88 ft/s)(3 s) = 264 ft
d. x = 0 ft + vt = (88 ft/s)(4 s) = 352 ft
e. x = 0 ft + vt = (88 ft/s)(5 s) = 440 ft
f. x = 0 ft + vt = (88 ft/s)(6 s) = 528 ft
h. The car is traveling at a constant speed and is moving in the positive direction. It moves the same distance in each one-second time interval.



2. A car is initially at rest. Its velocity changes by 10 mi/hr each second, ie its acceleration is (10mi/hr)/s. What is the car's velocity after:
a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. Draw a graphical representation of these results.
h. Describe the motion of the car.

Solution
2.
a. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((10mi/hr)/s)(1 s) = 10 mi/hr
b. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((10mi/hr)/s)(2 s) = 20 mi/hr
c. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((10mi/hr)/s)(3 s) = 30 mi/hr
d. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((10mi/hr)/s)(4 s) = 40 mi/hr
e. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((10mi/hr)/s)(5 s) = 50 mi/hr
f. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((10mi/hr)/s)(6 s) = 60 mi/hr
h. The car is moving faster as time passes. Its speed is increasing by 10 mi/hour every second. It is always traveling in the positive direction.



3. A car is initially at a position of x = 0. It is traveling in the negative direction at a speed of 88 ft/s - in other words it is traveling at a velocity of -88ft/s.. What is the car's position after:

a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. Draw a graphical representation of these results.
i. Describe the motion of the car.

Solution:

3.

a. $x_f = x_i + vt$

$x = 0\text{ ft} + vt = (-88\text{ ft/s})(1\text{ s}) = -88\text{ ft}$

b. $x = 0\text{ ft} + vt = (-88\text{ ft/s})(2\text{ s}) = -176\text{ ft}$

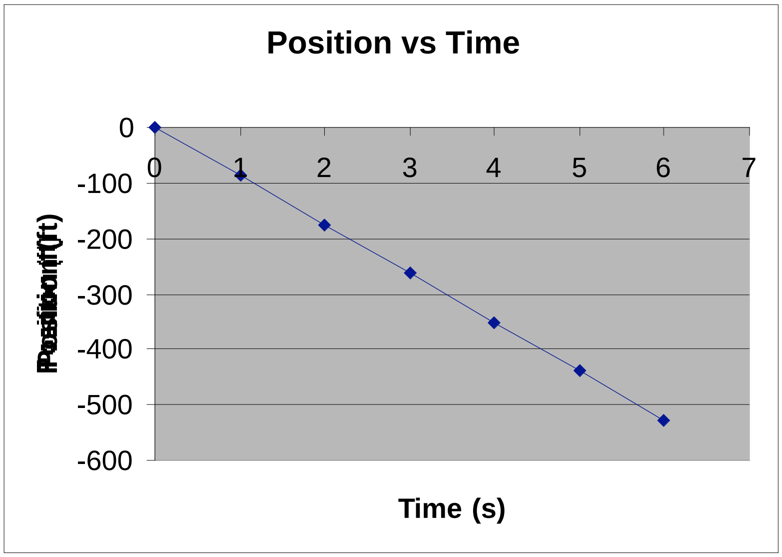
c. $x = 0\text{ ft} + vt = (-88\text{ ft/s})(3\text{ s}) = -264\text{ ft}$

d. $x = 0\text{ ft} + vt = (-88\text{ ft/s})(4\text{ s}) = -352\text{ ft}$

e. $x = 0\text{ ft} + vt = (-88\text{ ft/s})(5\text{ s}) = -440\text{ ft}$

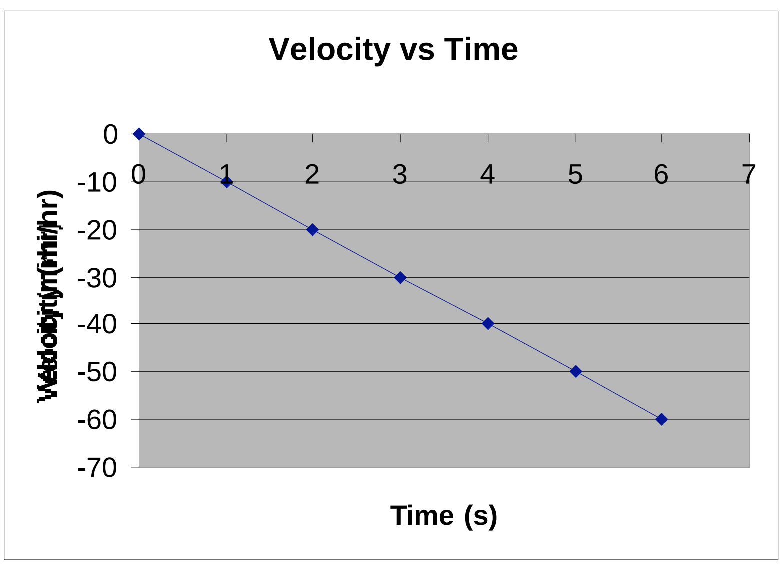
f. $x = 0\text{ ft} + vt = (-88\text{ ft/s})(6\text{ s}) = -528\text{ ft}$

i. The car is traveling at a constant speed and is moving in the negative direction. It moves the same distance in each one-second time interval.



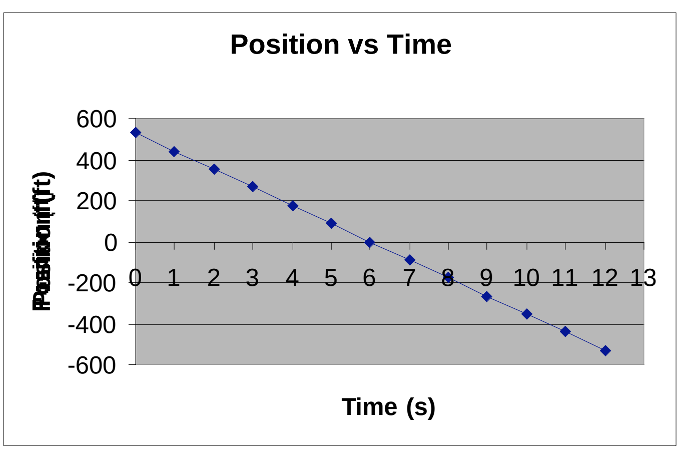
4. A car is initially at rest. Its speed changes by -10 mi/hr each second, ie its acceleration is (-10mi/hr)/s. What is the car's velocity after:
a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. Draw a graphical representation of these results.
h. Describe the motion of the car.

Solution
4.
a. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((-10mi/hr)/s)(1 s) = -10 mi/hr
b. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((-10mi/hr)/s)(2 s) = -20 mi/hr
c. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((-10mi/hr)/s)(3 s) = -30 mi/hr
d. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((-10mi/hr)/s)(4 s) = -40 mi/hr
e. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((-10mi/hr)/s)(5 s) = -50 mi/hr
f. $v_f = v_i + (a)$ (delta t)
$v = 0$ mi/hr + ((-10mi/hr)/s)(6 s) = -60 mi/hr
h. The car is moving faster as time passes. Its speed is increasing by 10 mi/hour every second. It is always traveling in the negative direction.



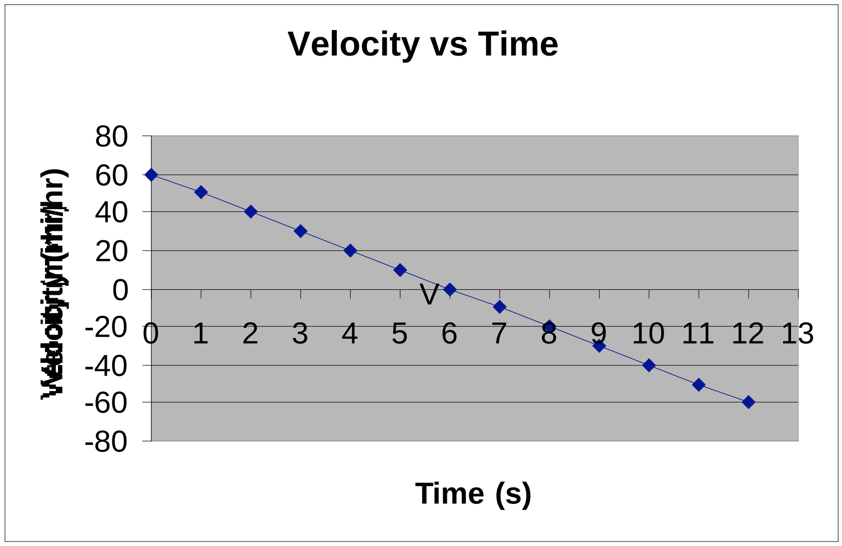
5. A car is initially at a position of x=528 ft.. It is traveling with a velocity of -88 ft/s. What is the car's position after:
a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. 7 s
h. 8 s
i. 9 s
j. 10 s
k. 11 s
l. 12 s
m. Draw a graphical representation of these results.
n. Describe the motion of the car.

Solution:
5.
a. $x_f = x_i + vt$
$x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(1\ s) = 440\ ft$
b. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(2\ s) = 352\ ft$
c. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(3\ s) = 264\ ft$
d. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(4\ s) = 176\ ft$
e. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(5\ s) = 88\ ft$
f. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(6\ s) = 0\ ft$
g. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(7\ s) = -88\ ft$
h. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(8\ s) = -176\ ft$
i. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(9\ s) = -264\ ft$
j. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(10\ s) = -352\ ft$
k. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(11\ s) = -440\ ft$
l. $x = 528\ ft + vt = 528\ ft + (-88\ ft/s)(12\ s) = -528\ ft$
n. The car starts out at the 528 ft mark and then passes the origin and moves into negative territory. It travels at a constant speed and it is always moving in the negative direction. It moves the same distance in each one-second time-interval.



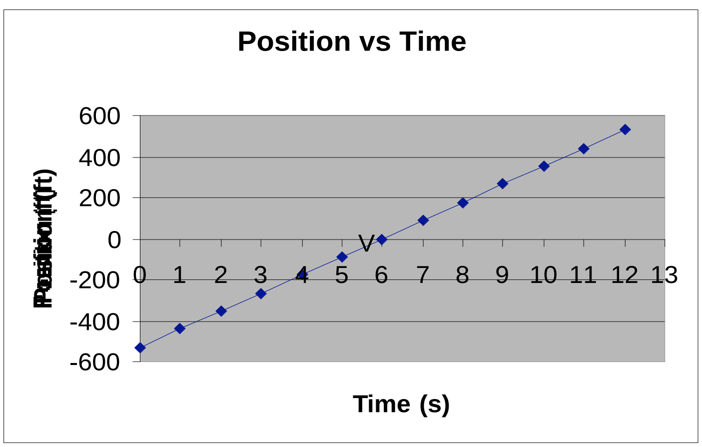
6. A car is initially traveling at 60 mi/hr. Its velocity changes by -10 mi/hr each second, ie its acceleration is (-10mi/hr)/s. What is the car's velocity after:
a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. 7 s
h. 8 s
i. 9 s
j. 10 s
k. 11 s
l. 12 s
m. Draw a graphical representation of these results.
n. Describe the motion of the car.

Solution
6.
a. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(1 s) = 50 mi/hr
b. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(2 s) = 40 mi/hr
c. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(3 s) = 30 mi/hr
d. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(4 s) = 20 mi/hr
e. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(5 s) = 10 mi/hr
f. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(6 s) = 0 mi/hr
g. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(7 s) = -10 mi/hr
h. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(8 s) = -20 mi/hr
i. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(9 s) = -30 mi/hr
j. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(10 s) = -40 mi/hr
k. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(11 s) = -50 mi/hr
l. $v_f = v_i + (a)$ (delta t)
 v = 60 mi/hr + ((-10mi/hr)/s)(12 s) = -60 mi/hr
n. The car is slowing down until it stops. Then it starts to speed up, traveling in the opposite direction. It ends up traveling in the negative direction.


Velocity vs Time
80
60
40
20
0
-20
-40
-60
-80
0 1 2 3 4 5 6 7 8 9 10 11 12 13
V
Time (s)

7. A car is initially at a position of x = -528 ft. It is at a velocity of 88 ft/s. What is the car's position after:
a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. 7 s
h. 8 s
i. 9 s
j. 10 s
k. 11 s
l. 12 s
m. Draw a graphical representation of these results.
n. Describe the motion of the car.

Solution:
7.
a. $x_f = x_i + vt$
$x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(1\text{ s}) = -440\text{ ft}$
b. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(2\text{ s}) = -352\text{ ft}$
c. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(3\text{ s}) = -264\text{ ft}$
d. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(4\text{ s}) = -176\text{ ft}$
e. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(5\text{ s}) = -88\text{ ft}$
f. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(6\text{ s}) = 0\text{ ft}$
g. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(7\text{ s}) = 88\text{ ft}$
h. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(8\text{ s}) = 176\text{ ft}$
i. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(9\text{ s}) = 264\text{ ft}$
j. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(10\text{ s}) = 352\text{ ft}$
k. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(11\text{ s}) = 440\text{ ft}$
l. $x = -528\text{ ft} + vt = -528\text{ ft} + (88\text{ ft/s})(12\text{ s}) = 528\text{ ft}$
n. The car starts out at the -528 ft mark and then passes the origin and moves into positive territory. It travels at a constant speed and it is always moving in the positive direction. It moves the same distance in each one-second time-interval.



8. A car is initially traveling at -60 mi/hr. Its speed changes by 10 mi/hr each second, ie its acceleration is (10mi/hr)/s. What is the car's speed after:

a. 1 s
b. 2 s
c. 3 s
d. 4 s
e. 5 s
f. 6 s
g. 7 s
h. 8 s
i. 9 s
j. 10 s
k. 11 s
l. 12 s
m. Draw a graphical representation of these results.
n. Describe the motion of the car.

Solution

8.

a. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(1 s) = -50 mi/hr

b. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(2 s) = -40 mi/hr

c. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(3 s) = -30 mi/hr

d. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(4 s) = -20 mi/hr

e. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(5 s) = -10 mi/hr

f. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(6 s) = 0 mi/hr

g. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(7 s) = 10 mi/hr

h. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(8 s) = 20 mi/hr

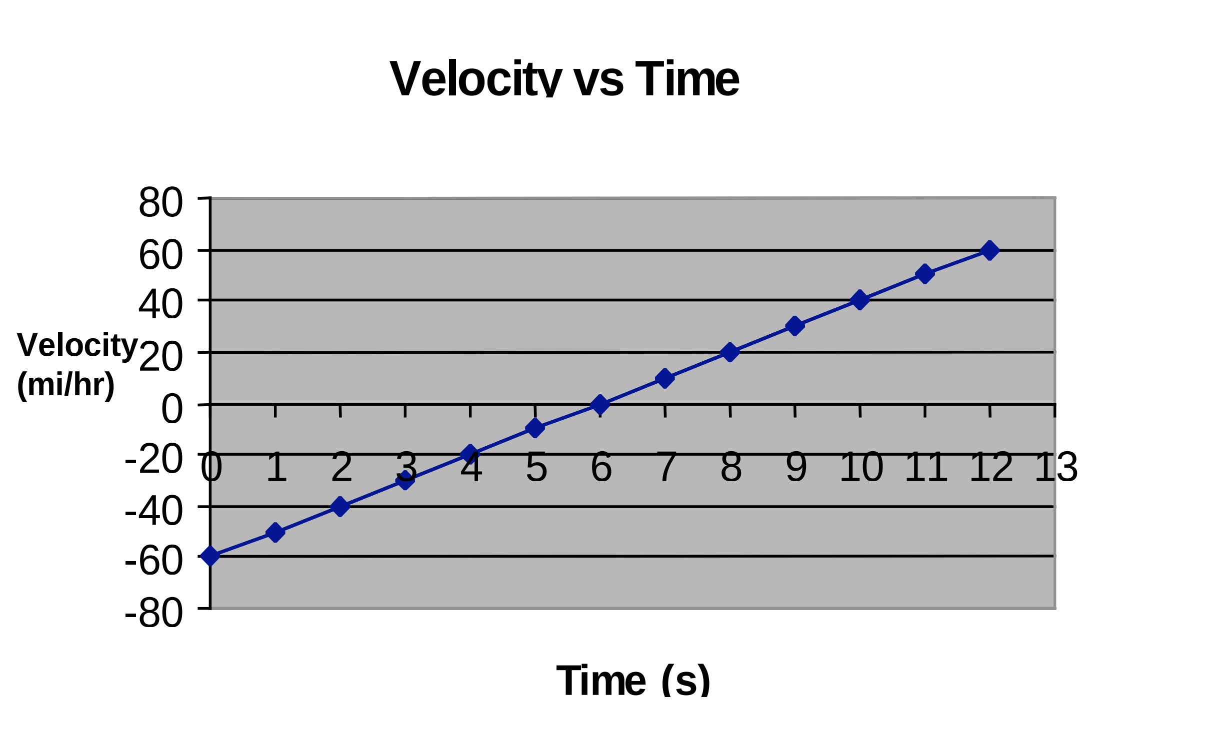
i. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(9 s) = 30 mi/hr

j. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(10 s) = 40 mi/hr

k. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(11 s) = 50 mi/hr

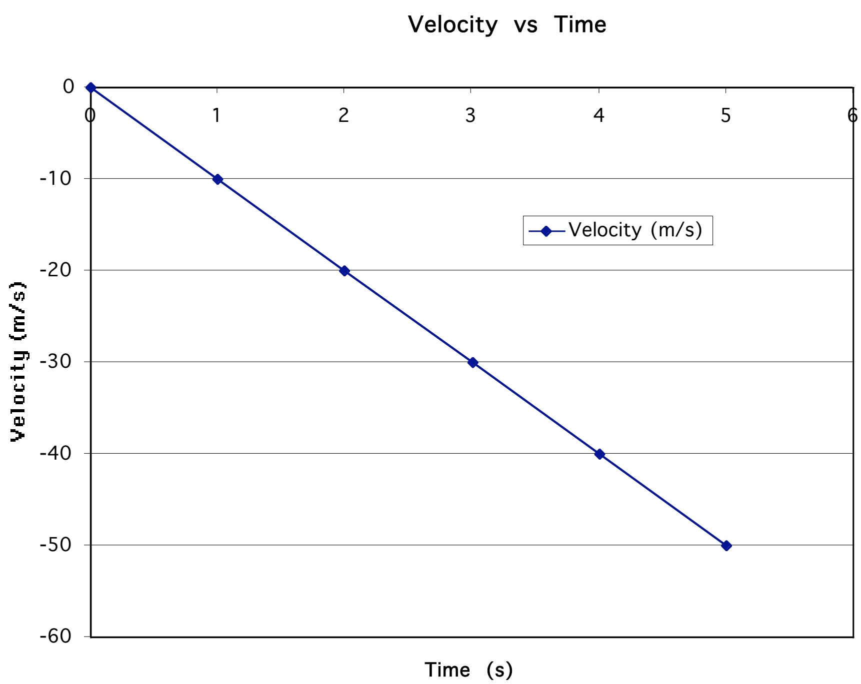
l. $v_f = v_i$ + (a) (delta t)
v = -60 mi/hr + ((10mi/hr)/s)(12 s) = 60 mi/hr

n. The car is slowing down until it stops. Then it starts to speed up, traveling in the opposite direction. It ends up traveling in the positive direction.


Velocity vs Time
80
60
40
20
0
-20
-40
-60
-80
Velocity (mi/hr)
0 1 2 3 4 5 6 7 8 9 10 11 12 13
Time (s)

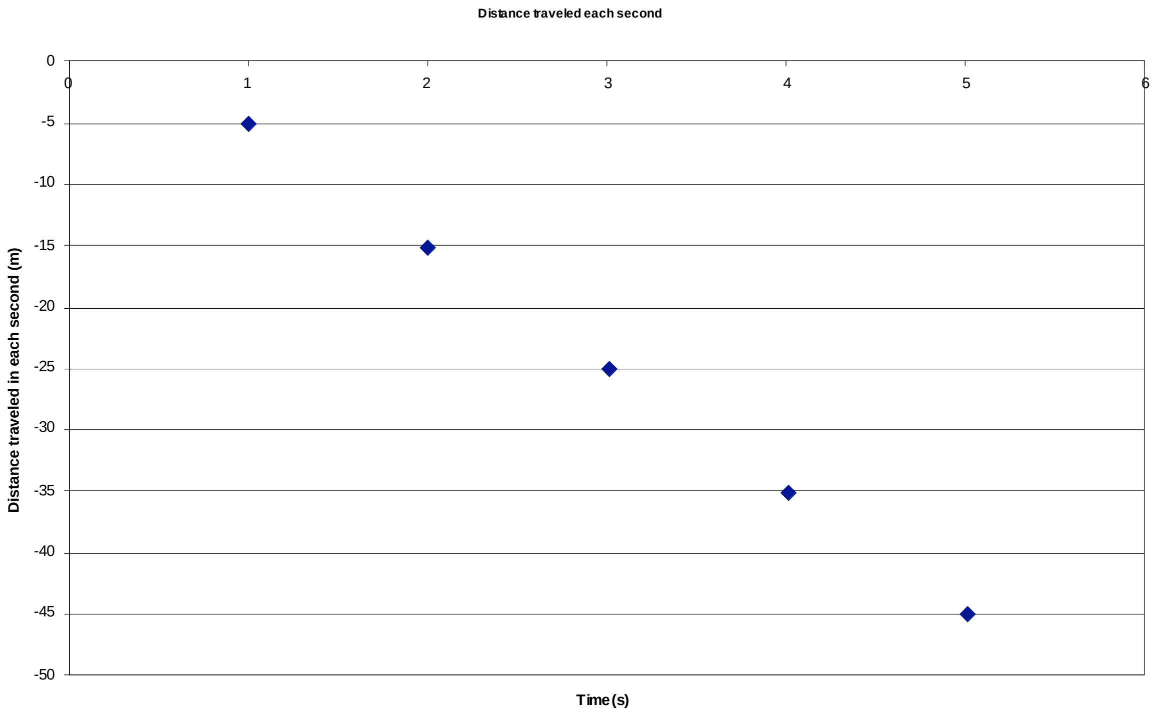
9. A skydiver steps off a tall cliff. Her speed changes by -10m/s every second. i.e. her acceleration is (-10m/s)/s. What is her speed after:
a. 0 s
b. 1 s
c. 2 s
d. 3 s
e. 4 s
f. 5 s
g. Draw a graphical representation of these results.
h. Describe the motion of the skydiver.

Solution
9.
a. $v_f = v_i + (a)\,(\text{delta t})$
   v = 0 m/s + ((-10m/s)/s)(0 s) = 0 m/s
b. $v_f = v_i + (a)\,(\text{delta t})$
   v = 0 m/s + ((-10m/s)/s)(1 s) = -10 m/s
c $v_f = v_i + (a)\,(\text{delta t})$
   v = 0 m/s + ((-10m/s)/s)(2 s) = -20 m/s
d. $v_f = v_i + (a)\,(\text{delta t})$
   v = 0 m/s + ((-10m/s)/s)(3 s) = -30 m/s
e. $v_f = v_i + (a)\,(\text{delta t})$
   v = 0 m/s + ((-10m/s)/s)(4 s) = -40 m/s
f. $v_f = v_i + (a)\,(\text{delta t})$
   v = 0 m/s + ((-10m/s)/s)(5 s) = -50 m/s
h. The skydiver starts from rest and then picks up speed as time goes on, moving faster and faster. The skydiver is always falling towards the earth.



For the skydiver in problem 9, calculate how far she travels each second and the direction of travel - use the average speed for each one second interval.

aa. Between 0 and 1 s: delta x = v_{avg} x 1 s = (0m/s + -10m/s)/2 x 1 s = -5m (downward)
bb Between 1 and 2 s: delta x = v_{avg} x 1 s = (-10m/s + -20m/s)/2 x 1 s = -15m (downward)
cc Between 2 and 3 s: delta x = v_{avg} x 1 s = (-20m/s + -30m/s)/2 x 1 s = -25m (downward)
dd. Between 3and 4 s: delta x = v_{avg} x 1 s = (-30m/s + -40m/s)/2 x 1 s = -35m (downward)
ee. Between 4 and 5 s: delta x = v_{avg} x 1 s = (-40m/s + -50m/s)/2 x 1 s = -45m (downward)
ff. Plot the average velocity of the skydiver at 1-second intervals.



For the skydiver in problem 9, calculate the distance below the cliff edge that the skydiver has fallen after each 1 s interval.

Call the position of the cliff at x=0

aaa. After 1 s: Height = x initial + delta x = 0m - 5m = -5 m

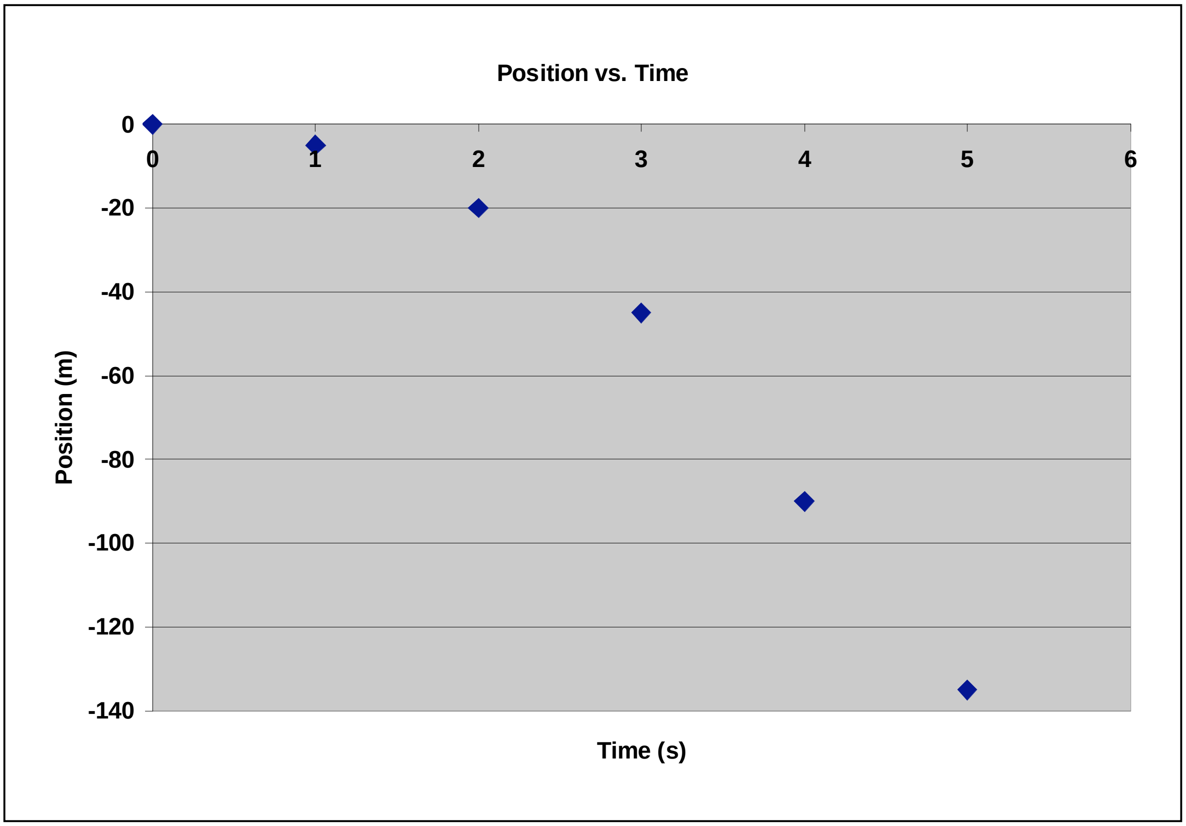
bbb. After 2 s: Height = x initial + delta x = -5m - 15m = -20 m

ccc. After 3 s: Height = x initial + delta x = -20m - 25m = -45 m

ddd. After 4 s: Height = x initial + delta x = -45m - 35m = -90 m

eee. After 5 s: Height = x initial + delta x = -90m - 45m = -135 m

kkk. Plot the position of the skydiver below the cliff edge in 1 second intervals.



10. A toy rocket is launched from the ground with an initial velocity upward of 50m/s. Its velocity changes by (-10m/s) every second, i.e. its acceleration is (-10m/s)/s. What is its speed after:

a. 0 s
b. 1 s
c. 2 s
d. 3 s
e. 4 s
f. 5 s
g. 6 s
h. 7 s
i. 8 s
j. 9 s
k. 10 s
l. Draw a graphical representation of these results.
m. Describe the motion of the toy rocket.

Solution:

10.

a. $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(0 s) = 50 m/s

b. $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(1 s) = 40 m/s

c $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(2 s) = 30 m/s

d. $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(3 s) = 20 m/s

e. $v_f = v_i + (a)$ (delta t)
   v = 50 mi/hr + ((-10m/s)/s)(4 s) = 10 m/s

f. $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(5 s) = 0 m/s

g. $v_f = v_i + (a)$ (delta t)
   v = 50 m/hr + ((-10m/s)/s)(6 s) = -10 m/s

h. $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(7 s) = -20 m/s

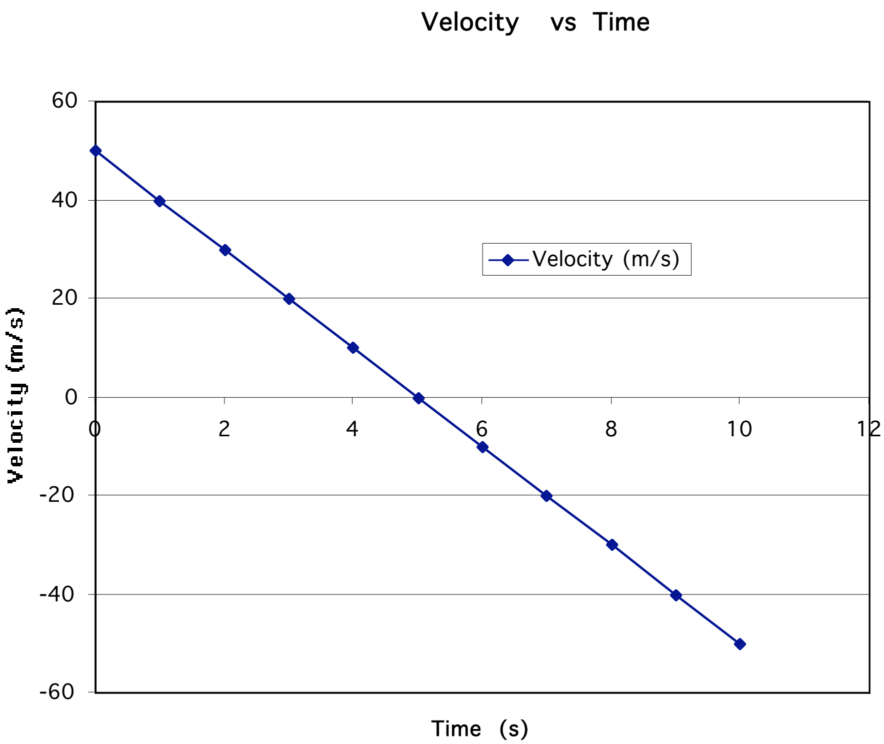
i $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(8 s) = -30 m/s

j. $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(9 s) = -40 m/s

k. $v_f = v_i + (a)$ (delta t)
   v = 50 m/s + ((-10m/s)/s)(10 s) = -50 m/s

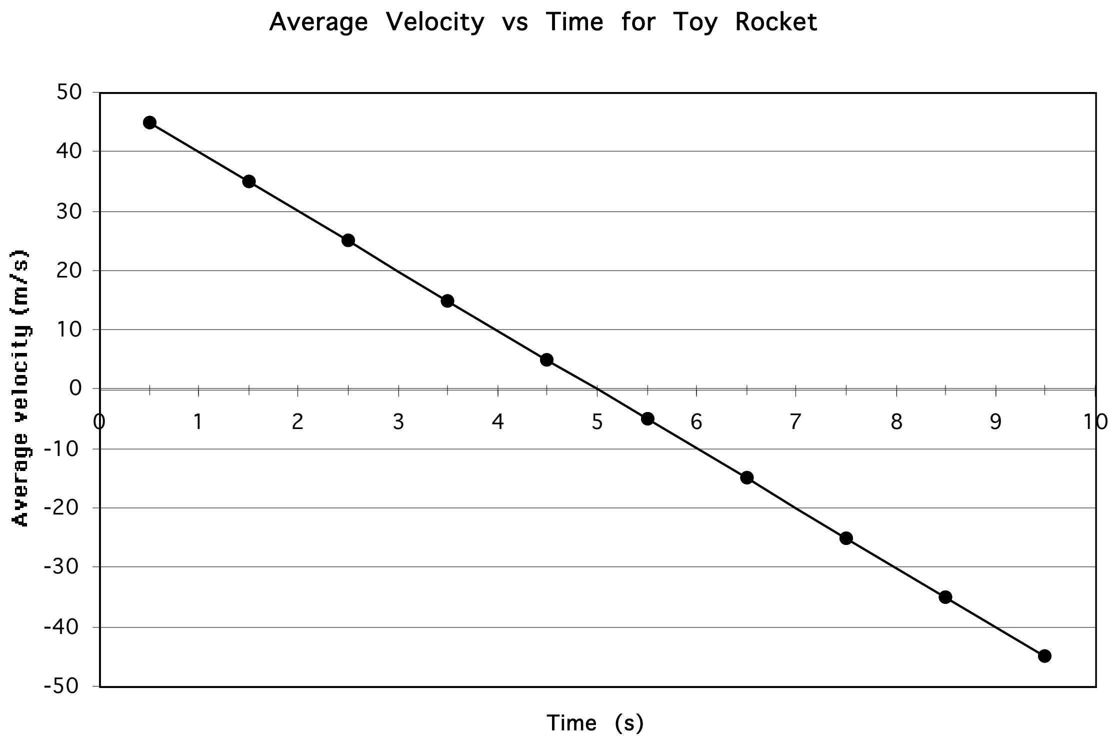
m. The toy rocket is moving upwards, but is slowing down until it stops. Then it starts to speed up, traveling downward towards the earth.

l.


Velocity vs Time
60
40
20
0
-20
-40
-60
0
2
4
6
8
10
12
Velocity (m/s)
Velocity (m/s)
Time (s)

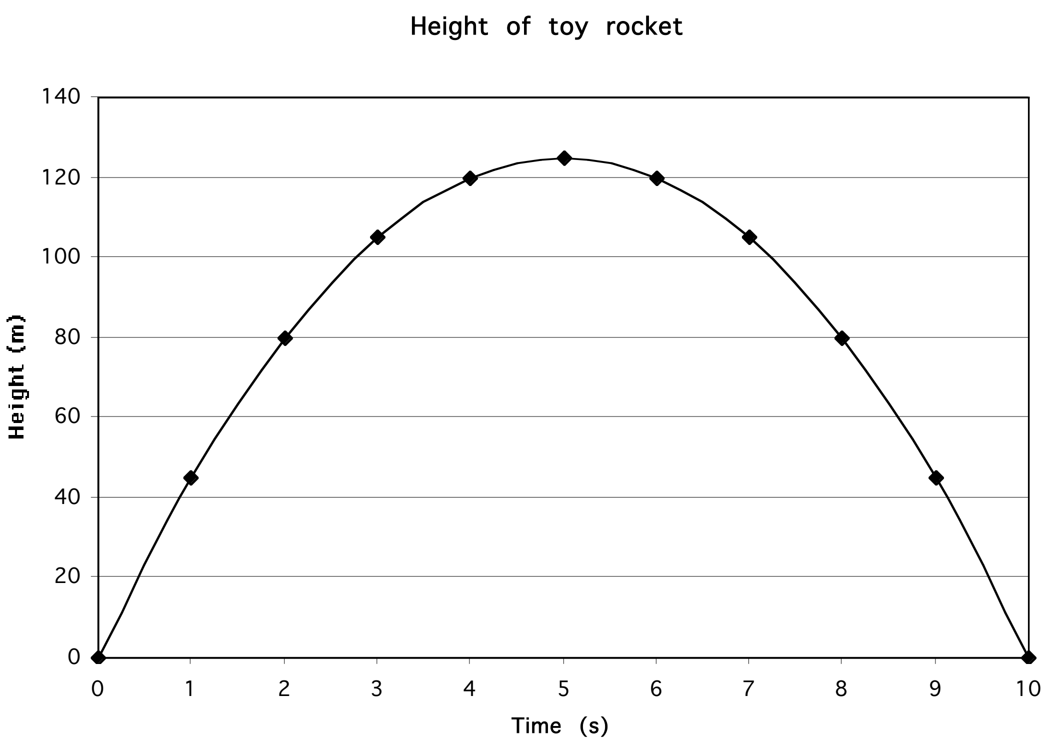
For the toy rocket in problem 10, calculate how far it travels each second and the direction of travel - use the average speed for each one second interval.

aa. Between 0 and 1 s: delta x = v_{avg} x 1 s = (50m/s + 40m/s)/2 x 1 s = 45m upward
bb Between 1 and 2 s: delta x = v_{avg} x 1 s = (40m/s + 30m/s)/2 x 1 s = 35m (upward)
cc Between 2 and 3 s: delta x = v_{avg} x 1 s = (30m/s + 20m/s)/2 x 1 s = 25m (upward)
dd. Between 3and 4 s: delta x = v_{avg} x 1 s = (20m/s + 10m/s)/2 x 1 s = 15m (upward)
ee. Between 4 and 5 s: delta x = v_{avg} x 1 s = (10m/s + 0m/s)/2 x 1 s = 5m (upward)
ff. Between 5 and 6 s: delta x = v_{avg} x 1 s = (0m/s - 10m/s)/2 x 1 s = -5m (downward)
gg. Between 6 and 7 s: delta x = v_{avg} x 1 s = (-10m/s - 20m/s)/2 x 1 s = -15m (downward)
hh. Between 7 and 8 s: delta x = v_{avg} x 1 s = (-20m/s - 30m/s)/2 x 1 s = -25m (downward)
ii. Between 8 and 9 s: delta x = v_{avg} x 1 s = (-30m/s - 40m/s)/2 x 1 s = -35m(downward)
jj. Between 9 and 10 s: delta x = v_{avg} x 1 s = (-40m/s - 50m/s)/2 x 1 s = -45m (downward)
kk. Plot the velocity of the rocket at 1 second intervals.



For the toy rocket in problem 10, calculate the height of the rocket after each 1 s interval.

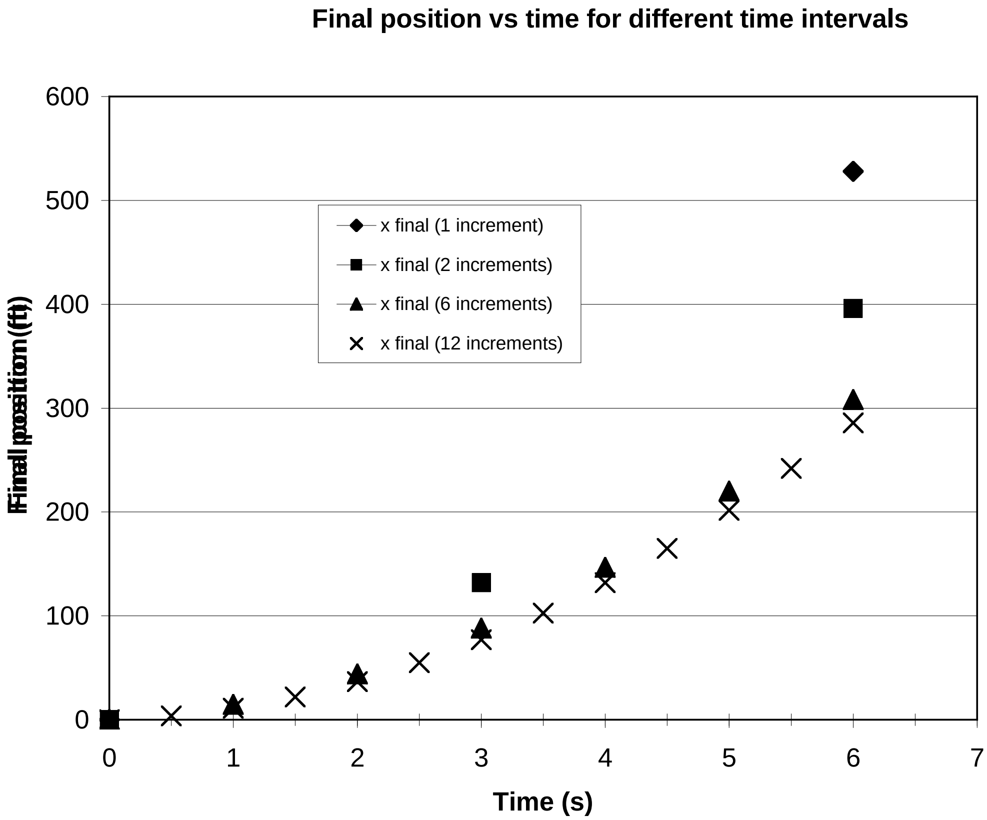
aaa. After 1 s: Height = x initial + delta x = 0m + 45m = 45 m
bbb. After 2 s: Height = x initial + delta x = 45m + 35m = 80 m
ccc . After 3 s: Height = x initial + delta x = 80m + 25m = 105 m
ddd. After 4 s: Height = x initial + delta x = 105m + 15m = 120 m
eee. After 5 s: Height = x initial + delta x = 120m + 5m = 125 m
fff. After 6 s: Height = x initial + delta x = 125m - 5m = 120 m
ggg. After 7 s: Height = x initial + delta x = 120m - 15m = 105 m
hhh. After 8 s: Height = x initial + delta x = 105m - 25m = 80 m
iii. After 9 s: Height = x initial + delta x = 80m - 35m = 45 m
jjj. After 10 s: Height = x initial + delta x = 45m - 45m = 0 m
kkk. Plot the height of the rocket in 1 second intervals.



11. Let's investigate the relationship between position, speed, and acceleration in a very straightforward way. Recall that for an object moving with constant acceleration, the speed changes by the same amount each time period that you choose. And for constant speed, the distance traveled by an object is the same in each time period that you choose.
So figure out the acceleration, speed change, final speed, distance traveled and final distance for a situation with decreasing time intervals. Assume that a car is accelerating at from 0 to 60 mi/hr in 6 sec. Recall that 60 mi/hr is 88 ft/s. Make a spreadsheet chart of these parameters using the following number of time intervals:
a. 1
b. 2
c. 6
d. 12
e. Make a graph of the position of the car vs time for each number of time intervals.
f. What do you notice about the graph? Is it linear or quadratic? Does it seem to converge to a certain curve as the time intervals get shorter and shorter?
g. Can you demonstrate that there is a mathematical relationship between distance traveled at constant acceleration for a given interval of time and the data you generated in this table?
Answers a-d in table below.

t (s)	delta t (s)	delta v (ft/s)	v final (ft/s)	delta x (ft)	x final (ft)
	(=6 s/increments)	(=(88ft/s)/increments)	(= v previous + delta v)	(=v final * delta t)	(= x previous + delta x)
	1 increment				
0					
6	6	88	88	528	528
	2 increments				
0					
3	3	44	44	132	132
6	3	44	88	264	396
	6 increments				
0					
1	1	14.67	14.67	14.67	14.674
2	1	14.67	29.34	29.34	44.014
3	1	14.67	44.01	44.01	88.024
4	1	14.67	58.68	58.68	146.704
5	1	14.67	73.35	73.35	220.054
6	1	14.67	88.02	88.02	308.074
	12 increments				
0					
0.5	0.5	7.33	7.33	3.665	3.665
1	0.5	7.33	14.66	7.33	10.995
1.5	0.5	7.33	21.99	10.995	21.99
2	0.5	7.33	29.32	14.66	36.65
2.5	0.5	7.33	36.65	18.325	54.975
3	0.5	7.33	43.98	21.99	76.965
3.5	0.5	7.33	51.31	25.655	102.62
4	0.5	7.33	58.64	29.32	131.94
4.5	0.5	7.33	65.97	32.985	164.925
5	0.5	7.33	73.3	36.65	201.575
5.5	0.5	7.33	80.63	40.315	241.89
6	0.5	7.33	87.96	43.98	285.87

e.



f. The graph appears to be quadratic, not linear.

g. $x = 0.5\ a\ t^2 = 0.5\ (10\text{mi}/(\text{hr-s}))\ (6\text{s})^2$
$= 180\text{mi-s/hr} \times (1\text{hr}/3600\text{s}) \times (5280\text{ft/mi})$
$= 264\ \text{ft}$

This is very close to the value of 286 ft that we obtained using 12 increments of time in part d.

12. Show that $x = v_i t + 0.5*at^2$ is equivalent to $v_f^2 - v_i^2 = 2ax$. Recall that $v_f = v_i + at$, so $t = (v_f - v_i)/a$

$x = v_i t + 0.5*at^2$

$= v_i (v_f - v_i)/a + 0.5a(v_f - v_i)^2/a^2$

$= (v_f - v_i) \{v_i/a + 0.5(v_f - v_i) /a\}$

$= (v_f - v_i) \{0.5(v_f + v_i)/a\}$

$= 0.5 (v_f^2 - v^2_i)/a$

So $v_f^2 - v_i^2 = 2ax$

13. Begin with the equation $v_f^2 - v_i^2 = 2ax$.

a. Multiply both sides of the equation by 0.5m, where m is the mass of an object moving with speed v.

$0.5mv_f^2 - 0.5mv_i^2 = max$

b. Now define the initial kinetic energy of the object as $0.5mv_i^2$
and define the final kinetic energy of the object as $0.5mv_f^2$
and define the force on the object as ma
and recall that x is the distance traveled by the object.
Rewrite the equation in part a using these definitions.

final kinetic energy - initial kinetic energy = force * distance

Since force*distance = work, this shows that work is equal to the change in kinetic energy.

14. Again consider the equation $0.5mv_f^2 - 0.5mv_i^2 = max$ from problem 13.

a. Now consider an object free falling near the earth and call this constant acceleration g. Let the distance the object falls be called h. Rewrite the above equation.

final kinetic energy - initial kinetic energy = mgh

b. Now define a term called potential energy difference as mgh. Rewrite the above equation.

final kinetic energy - initial kinetic energy = potential energy difference or (final kinetic energy - initial kinetic energy) - potential energy difference = 0

or (change in kinetic energy) - (change in potential energy) = 0 : Energy Conservation!

Note that the law of energy conservation is a direct consequence of the time translational symmetry of nature. In other words, since nature's laws work the same today as they did yesterday or will tomorrow, then as a result we get the law of the conservation of energy. For further details about the relationship between symmetry and conservations laws, see the wonderful web site: http://www.emmynoether.com/noeth.htm

15. Consider the possible paths A, B, C, D an object may take to get from one point to another point in the same amount of time - in this case 10 m from the starting point in 10 seconds. It turns out that it will follow the path that has the minimum value of the sum of the (product of the kinetic energy ($0.5mv^2$) and the time interval) over the path. This value is called the "action." (We are neglecting gravity. If gravity is present, we must include the potential energy in some way.) Assume a mass of 1kg. The “Principle of Least Action” is another way of formulating the laws of motion in the same way that the Principle of Least Time (also known as Fermat's Principle) is another way of formulating the laws governing the behavior of light.

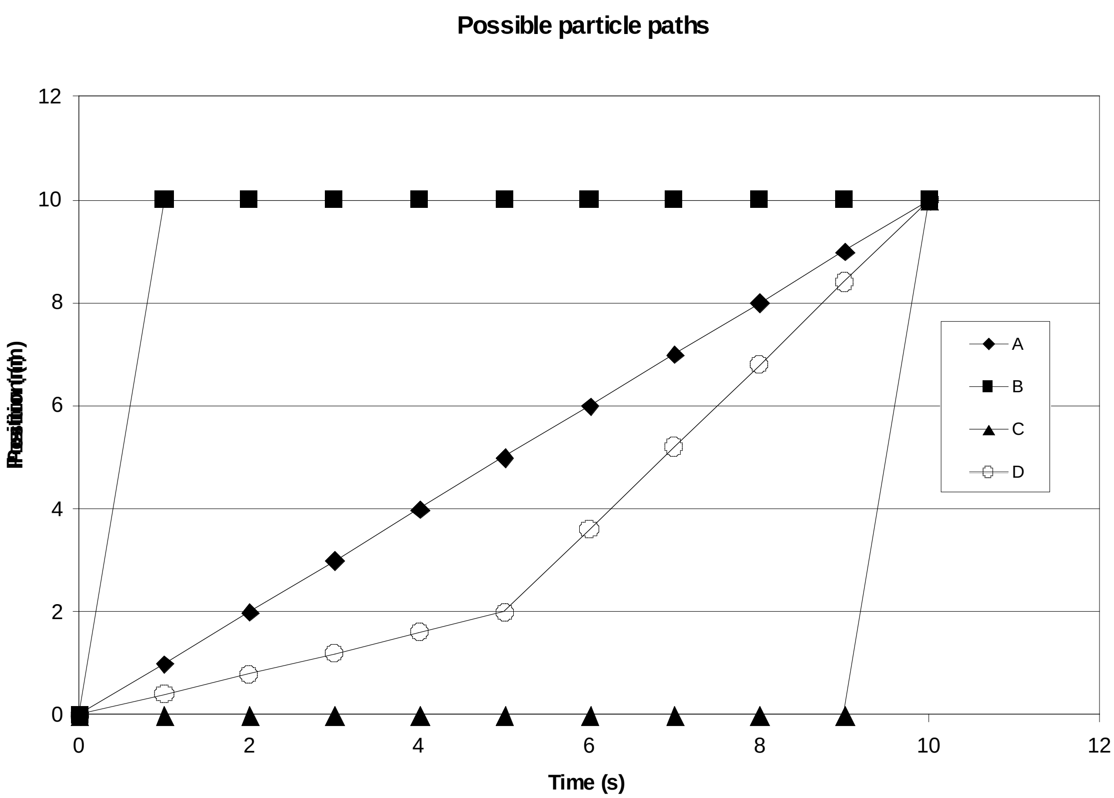
a. Calculate the “action" for each of the 4 paths. Which path will the object follow?
Path A: Action = 0.5 (1kg) $(1m/s)^2$ * 10 s = 5 kg m^2/s

Path B: Action = 0.5 (1kg) $(10m/s)^2$ * 1s = 50 kg m^2/s

Path C: Action = 0.5 (1kg) $(10m/s)^2$ * 1s = 50 kg m^2/s

Path D: Action = 0.5 (1kg) $(2m/5s)^2$ * 5 s + 0.5 (1kg) $(8m/5s)^2$ * 5 s = 6.8 kg m^2/s

The object will follow path A, because that is the path of Least Action. It is the path of an object moving at constant speed.
(For a beautiful discussion of this topic - for advanced students - see the Feynman Lectures on Physics, Volume 1, Chapter 19: "The Principle of Least Action.")



b. Describe the motions of the objects traveling along these "paths."

A: This object travels at a constant speed of 1m/s from 0 to 10 s.

B. This object travels 10 m in 1 s, and then stops.

C. This object does not move for 9 s, and then travels 10m in 1s.

D. This object travels 2 m in 5 s, then 8 m in 5 s, so it moves faster during the time interval of 5-10 s.

Investigation #6 – Driving Safety

1. a. How far can a car travelling at 60 mi/h go in 0.1 s?
   b. How far can a car travelling at 60 mi/h go in 0.5 s?
   c. How far can a car travelling at 60 mi/h go in 1.0 s?
   d. How far can a car travelling at 60 mi/h go in 2.0 s?
   e. How far can a car travelling at 60 mi/h go in 5.0 s?

For c, d, and e, use your scale model car (Car A) and map (Map A) to simulate these situations.

2. Travelling at 60 mi/h,

   (a) Estimate how far you will travel when you turn around to talk to a friend in the back seat. Assume it takes 2 s for this to happen.
   (b) Estimate how far you will travel when you search for a CD in the glove compartment. Assume it takes 1 s for this to happen.
   (c) Estimate how far you will travel when you turn to the side to see if the space next to you is clear for passing.
   (d) The low beams of your headlights will allow you to see about 160 feet in front of you at night. How long does it take your car to travel this distance?
   (e) Most drivers need about 1.5 s to react to a new situation. How far will your car travel in this time interval?
   (f) After the brakes are applied to a car traveling at 60 mi/h, the car needs about 227 feet to stop. Considering reaction time (about 1.5 s) and braking distance, how long a distance will it take to stop if you see a problem up ahead at night? Why does this show the dangers of driving at night?
   (g) What are the safety implications of these calculations?

For a - f, use your scale model car (Car A) and map (Map A)to simulate these situations.

3. When drunk, the reaction time of the average driver doubles from 1.5 s to 3.0 s.

   (a) How far will a drunk driver's car travel before it stops if it was traveling at 60 mi/hr? (Use the information provided in problem 2f.)
   (b) When talking on a cell phone, the reaction time of the average driver also doubles from 1.5 s to 3.0 s. How far will a "cell-phone-driver's" car travel before it stops if it was traveling at 60 mi/h?
   (c) What are the safety implications of these calculations?

For a and b, use the drunk driver scale model car (Car E) and map (Map A) to simulate these situations.

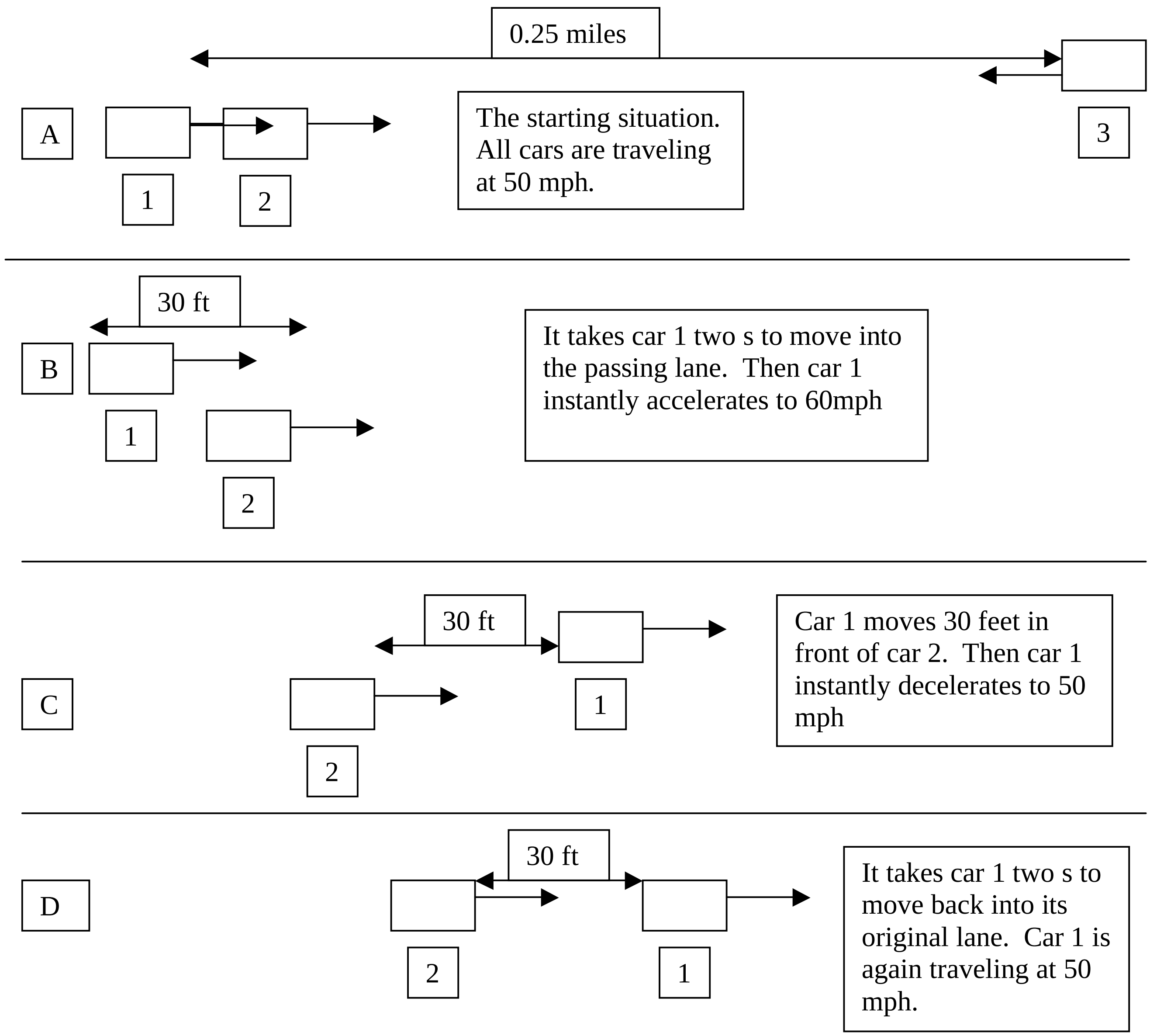
4. On a 2-lane road (one lane in each direction), you (car 1) decide to pass a car (car 2) in front of you that is traveling at 50 mi/h. You see another car (car 3) coming towards you from the other direction that is traveling at 50 mi/hr. When you are just behind car 2, you instantly accelerate to 60 mi/h and move into the other lane. You pass car 2 then move back into your original lane, and instantly decelerate back to 50 mph. Assume that it takes 2 s to travel into the passing lane and 2 s to move back into your original lane. Also assume that you will pass back into your lane when you are two complete car lengths in front of the car you are passing. Each car is 5 m long. The oncoming car is 0.25 miles away from car 1.

a. Will car 1 successfully pass car 2 without hitting the oncoming car 3?

b. If so, how many seconds later would your car and the oncoming car be at the same position on the road?

c. What lesson did you learn from this?

Use your scale model car B and map A to simulate these situations.


0.25 miles
A
1
2
3
The starting situation. All cars are traveling at 50 mph.
30 ft
B
1
2
It takes car 1 two s to move into the passing lane. Then car 1 instantly accelerates to 60mph
30 ft
C
1
2
Car 1 moves 30 feet in front of car 2. Then car 1 instantly decelerates to 50 mph
30 ft
D
2
1
It takes car 1 two s to move back into its original lane. Car 1 is again traveling at 50 mph.

5. Suppose you a traffic engineer working for the California Department of Transportation. Your job is to set the timing of traffic lights. You are also to only use metric units. How long should traffic lights be yellow for the following speeds: 45 km/hr, 65 km/hr, 85 km/hr, and 105 km/hr? Use the results of problem 5. Use car E and map A to simulate the effectiveness of your choice of yellow traffic light times.

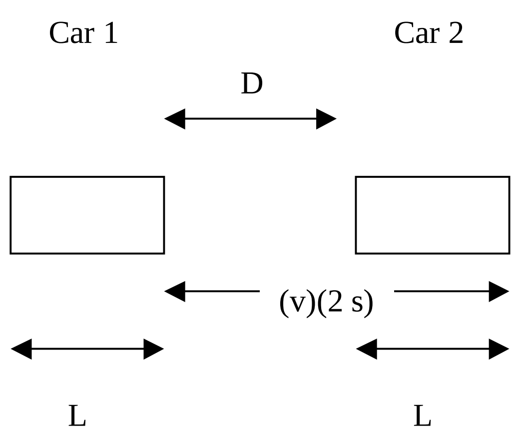
6. A general rule of thumb taught to drivers is to leave one car length of distance between cars for every 10 mph. Assume that a typical car length is 15 feet or 3 meters. Based on the results of problem 5, does this make sense? Why or why not? Consider 2 different cases.

a. You are traveling at 60 mph and suddenly the traffic in front of you slows to 50 mph.

b. You are traveling at 60 mph and suddenly the traffic in front of you slows to an abrupt stop. Why does this tend to lead to multi-car pile-ups?

Use your scale model car D and map A to simulate these situations.

7. Another way to judge appropriate spacing between cars when driving is to consider what is called "headway." Headway is defined as the elapsed time between the front of the lead vehicle passing a point on the roadway and the front of the following vehicle passing the same point. Most driving manuals recommend a headway of at least 2 s. How does a headway of 2-s compare to the rule of thumb that you should leave 1 car length between the front of your car and the back of the car in front of you for every 10-mph of speed?
Hint 1: Assume that each car length is 14.7 ft long. Recall that 10mph = 14.7 ft/s.
Hint 2: Assume two cars are moving at the same constant speed, one behind the other. Call the speed v. At time t = 0 the front of the lead vehicle passes a given point on the highway. Two seconds later the front of the second vehicle passes that same point. The total distance between the front of the two vehicles is therefore (v)(2 s).
Use your scale model car D and map A to simulate these situations.



8. You are driving at night to a friend's house. You make a sharp right hand turn onto another street and suddenly, 75 feet in front of you, you see someone crossing the street. Based on the results of problem 5, what is the maximum speed you could be traveling at and not hit the person. How fast should you be driving when making a turn onto a street at night?
Use your scale model car D and map A to simulate this situation.

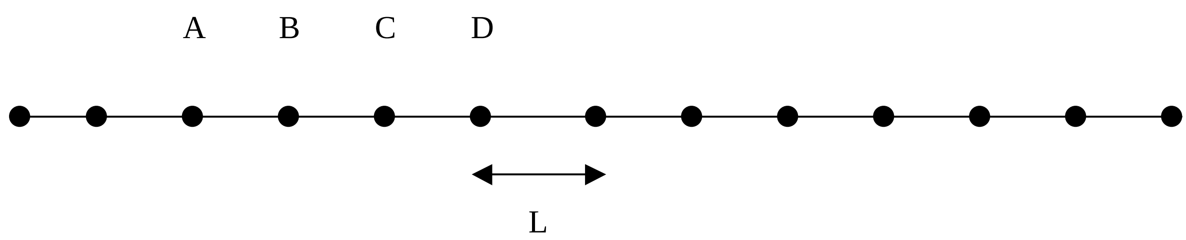
9. Headlights illuminate the road up to 160 feet in front of you. If you are on a road with stop signs, what is the fastest speed you can drive and still stop safely at night?
Use your scale model car D and map A to simulate this situation.

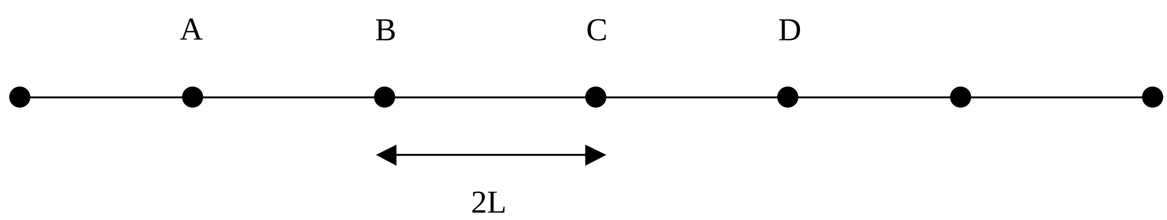
10. A fire engine is traveling at 25 m/s towards the Doppler bus station on its way to a fire. At its closest approach it passes right next to the station. Starting at 500 m before the station, it sends out a short blast of sound every 100 m. It stops sending these messages when it is 500 m past the station. Sound travels at 330 m/s. If you are standing at the Doppler bus station, determine the time interval between successive blasts of sound. Calculate and compare how the time intervals change when the fire engine is approaching you versus when it is moving away from you.

10A. A fire engine is traveling at 25 m/s on its way to a fire. At its closest approach it passes 100m from a bus station. Starting at 400 m before the station, it sends out a very short blast of sound every 100 m. It stops sending these messages when it is 400 m past the station. Sound travels at 330 m/s. If you are standing at the bus station, determine the time interval between successive blasts of sound. Calculate and compare (using a table and a chart) how the time intervals change when the fire engine is approaching you versus when it is moving away from you.

11. Jack and Jill each drive their vehicles 10,000 miles per year. Jack's vehicle has a fuel economy of 10 miles per gallon, Jill's 30 miles per gallon.
    a. How much fuel does each of them use in a year?
    b. How much fuel does the Jack and Jill household use in a year?
    c. How far do they travel in a year?
    d. What is their average household fuel economy? Is it the average of Jack's fuel economy and Jill's fuel economy?
    e. What would their average household fuel economy be if Jill's vehicle got 100 miles per gallon?
    f. What would their average household fuel economy be if Jill's vehicle got 1000 miles per gallon?
    g. What would their average household fuel economy be if Jill's vehicle got 10,000 miles per gallon?
    h. What would their average household fuel economy be if Jack's vehicle got 30 miles per gallon, the same as Jill's original vehicle?
    i. If you were in charge of making policy to reduce fuel consumption, what would you do?

12. Consider the following distribution of dots on the line below. Let's call the dots "galaxies" and let's call the line "the universe." Suppose that adjacent galaxies are all located a distance of L apart from each other in the universe. At a time T later, the universe has expanded a factor of two so that now all of the adjacent galaxies are a distance of 2L apart.
a. Suppose you are living in galaxy A. How fast does it appear that galaxies B, C, and D are receding from you?
b. Is there a correlation between the distance the galaxy is located from you and the speed with which it is receding from you. What is that relationship?
c. Do people in each galaxy see the same thing happening?





Investigation #6: Driving Safety Solutions

1. How far can a car travelling at 60 mi/h go in:

(a) 0.1 s?

$$\frac{60\text{ mi}}{\text{h}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{5280\text{ ft}}{1\text{ mi}} = \frac{88\text{ ft}}{\text{s}}$$

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 0.1\text{ s} = 8.8\text{ ft}$$

(b) 0.5 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 0.5\text{ s} = 44\text{ ft}$$

(c) 1 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 1.0\text{ s} = 88\text{ ft}$$

(d) 2 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 2.0\text{ s} = 176\text{ ft}$$

(e) 5 s?

$$x = v \times t = \frac{88\text{ ft}}{\text{s}} \times 5.0\text{ s} = 440\text{ ft}$$

2. Travelling at 60 mi/h,

(a) Estimate how far you will travel when you turn around to talk to a friend in the back seat. What are the safety implications of this?

Talking briefly to a friend in the back seat takes about 2 s. In 2 s, you will travel 176 feet (see 1 (d)). This is a long distance so that driving conditions might be drastically different compared to when you last looked at the road, possibly leading to a crash. It is best not to take your eyes off of the road

(b) Estimate how far you will travel when you search for a CD in the glove compartment. What are the safety implications of this?

Reaching and finding a CD in the glove compartment takes about 4 s. In this time, you will travel the following distance:

$$x = v \times t = \frac{88 \text{ ft}}{\text{s}} \times 4.0 \text{ s} = 352 \text{ ft}$$

This is a long distance so that driving conditions might be drastically different compared to when you last looked at the road, possibly leading to a crash. It is best not to take your eyes off of the road.

(c) Estimate how far you will travel when you turn to the side to see if the space next to you is clear for passing. What are the safety implications of this?

Looking to the side takes about 1 s. In 1 s, you will travel 88 feet (see 1 (c)). This is a long distance so that driving conditions might be drastically different compared to when you last looked at the road, possibly leading to a crash. It is best to minimize the time you take your eyes off of the road and it is prudent to leave much more than 88 feet between you and the car in front of you.

(d) The low beams of your headlights will allow you to see about 160 feet in front of you at night. How long does it take your car to travel this distance? What are the safety implications of this?

$$t = \frac{x}{v} = \frac{160 \text{ ft s}}{88 \text{ ft}} = 1.8 \text{ s}$$

You will travel the distance illuminated by your headlights in only 1.8 s. If an object is seen in your headlights, you have at most 1.8 s to avoid it if you stay travelling at 60 mph.

(e) Most drivers need about 1.5 s to react to a new situation. How far will your car travel in this time interval? What are the safety implications of this?

$$x = v \times t = \frac{88 \text{ ft}}{\text{s}} \times 1.5 \text{ s} = 132 \text{ ft}$$

Your car will travel 132 feet in an emergency situation before you even have a chance to hit the brakes!

> Teacher's Note: There is an interesting discussion of reaction times in the book Traffic Safety and the Driver. If you are anticipating an event, reaction times can be as short as 0.15 s. While driving, your reaction time is divided into a perception reaction time ("I need to brake") and a movement reaction time (movement of your foot). For drivers focusing on the car ahead of them, an average reaction time is 1.6 s. For drivers encountering an unexpected obstacle around a blind curve, an average reaction time is closer to 2.5 s.

(f) After the brakes are applied to a car traveling at 60 mi/h, the car needs about 227 feet to stop. Considering reaction time (about 1.5 s) and braking distance, how long a distance will it take to stop if you see a problem up ahead at night? Why does this show the dangers of driving at night?

The car will travel 132 ft while you are reacting to the situation (see 2e above) and then it will take 227 feet to stop. So to stop will require a total stopping distance of 132 ft + 227 ft = 359 ft. Since your headlights illuminate only the road 160 feet in front of you, at 60 mph, you will be unable to stop in time if an obstruction suddenly appears on the road in front of you at night.

Activity Suggestion: Have your students stand near a stoplight. They can determine a typical reaction time by measuring the time it takes for a car to begin moving after a stoplight turns from red to green.

3. How far will a drunk driver's car travel before it stops if it was traveling at 60 mi/hr? What are the safety implications of this? (Recall that the reaction time of a drunk driver is doubled compared to that of a sober driver – so the reaction time of a drunk driver is about 3.0 s)

The reaction time is 3.0 s so the distance the car travels at 60 mph in 3.0 s is:

$$x = v \times t = \frac{88 \text{ ft}}{\text{s}} \times 3.0 \text{ s} = 264 \text{ ft}$$

The braking distance is, as in 2 above, 227 ft.

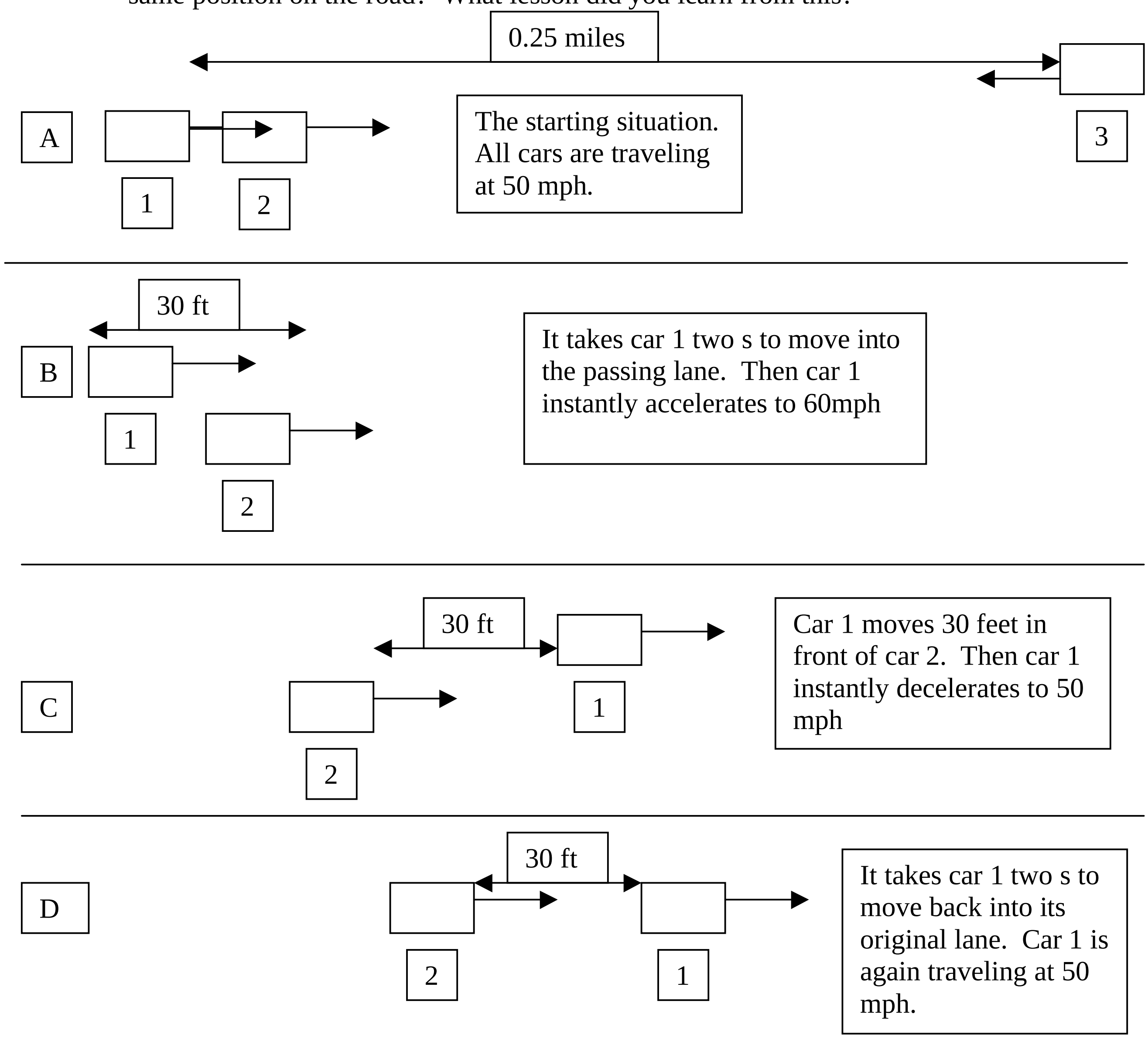
So the total braking distance is 264 ft + 227 ft = 491 ft.

The stopping distance of a drunk driver is 491 ft when traveling at 60 mph. This is (491ft - 359 ft) = 132 ft more than the stopping distance for a sober driver. A drunk driver reacts more slowly than a sober driver, so their car will travel a longer distance before it stops. If there is a problem in the road less than the stopping distance of 491 feet, the drunk driver will hit it. Since the reaction time of a driver on a cell phone is comparable to that of a drunk driver, the above analysis also holds for them.

4. On a 2-lane road (one lane in each direction), you (car 1) decide to pass a car (car 2) in front of you that is traveling at 50 mi/hr. You see another car (car 3) coming towards you from the other direction that is traveling at 50 mi/hr. When you are just behind car 2, you instantly accelerate to 60 mi/hr and move into the other lane. You pass car 2 then move back into your original lane, and instantly decelerate back to 50 mph. Assume that it takes 2 s to travel into the passing lane and 2 s to move back into your original lane. Also assume that you will pass back into your lane when you are two complete car lengths in front of the car you are passing. Each car is 5 m long. The oncoming car is 0.25 miles away from car 1.

a. Will car 1 successfully pass car 2 without hitting the oncoming car 3?

a. If so, how many seconds later would your car and the oncoming car be at the same position on the road? What lesson did you learn from this?



a. Car 1 must travel 60 feet more than car 2 to be able to pass it. Car 1 is travelling 10 mph (15 ft/s) faster than car 2 while it is in the passing lane. So car 1 will take the following amount of time to pass car 2.

$$t = \frac{x}{v} = \frac{60 \text{ ft s}}{15 \text{ ft}} = 4 \text{ s}$$

So car 1 takes 2 s to move into the passing lane (at 50 mph), 4 s to pass car 1 (at 60 mph), and then 2 more s to move back into its original lane (at 50 mph), the total time is 8 s.

In 8 s, car 1 will travels for 2 s at 50 mph, then for 4 s at 60 mph, then 2 s for 50 mph, so the total distance traveled by car 1 is:

$$x = \frac{73.3 \text{ ft}}{\text{s}} \times 2 \text{ s} + \frac{88 \text{ ft}}{\text{s}} \times 4 \text{ s} + \frac{73.3 \text{ ft}}{\text{s}} \times 2 \text{ s} = 645 \text{ ft}$$

In 8 s, car 3 will travel the following distance (towards car 1)

$$x = \frac{73.3 \text{ ft}}{\text{s}} \times 8 \text{ s} = 586 \text{ ft}$$

So at the end of 8 s car 1 and car 2 are 645 ft + 586 ft = 1231 ft closer. The cars started 0.25 miles apart, which is 1320 ft (recall that 1 mile = 5280 ft). When car 1 is back in its original lane after passing car 2, car 1 will be only 89 feet from car 3. Since their relative speed of approach is 50 mph + 50 mph = 100 mph = 147 ft/s, the safety margin for passing car 2 was less than 1 s.

b. Even though car 1 started to pass car 2 when car 3 was 0.25 miles away, car 1 and car 3 missed colliding at 50 mph by less than 1 s. So you must allow much more distance and time for passing than is obvious.

> Excerpt from Traffic Safety and the Driver (p. 118): "It is found that while drivers make reliable estimates of the distance to the oncoming car, they are insensitive to its speed... The inability of drivers to estimate oncoming speed leads them to decline safe passing opportunities when the oncoming car is travelling slower than expected, and to initiate unsafe passing maneuvers when the oncoming car is travelling faster than expected."

Note: As an extension, students could also plot the position vs. time and the velocity (or speed) vs. time for each of the three cars.

5. Suppose you a traffic engineer working for the California Department of Transportation. Your job is to set the timing of traffic lights. You are also to only use metric units. How long should traffic lights be yellow for the following speeds: 45 km/hr, 65 km/hr, 85 km/hr, and 105 km/hr? Use the results of problem 2 in the simulation tool development section.

First translate the metric speeds to English unit speeds.

a. $\frac{45 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 28 \text{ mph}$

b. $\frac{65 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 41 \text{ mph}$

c. $\frac{85 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 53 \text{ mph}$

d. $\frac{105 \text{ km}}{\text{h}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = 66 \text{ mph}$

Using the table of total stopping times in problem 2 in the simulation tool development section to estimate the total stopping times at the 4 speeds.

Speed (km/h)	Total stopping time (s) from problem 2 in simulation tool development section	California traffic manual suggested times for yellow lights
45	3.9	3.1
65	5.0	3.9
85	6.0	4.9
105	7.2	5.8

The yellow light times suggested by the California traffic manual seem to be about 1 second less than would be expected. Let's explore this further in the activities below.

Suggested student activity: Have your students measure the duration of yellow lights. For a certain speed limit, are they always set the same? How are they set at different speed limits?

e. What is the distance traveled by car during California department of transportation yellow light time if it stays at its initial speed.

At 28 mph:

$$\frac{28\,mi}{hr} \times \frac{88\,ft}{s} \frac{hr}{60\,mi} \times 3.1\,s = \frac{41.1\,ft}{s} \times 3.1\,s = 127\,ft$$

At 41 mph:

$$\frac{41\,mi}{hr} \times \frac{88\,ft}{s} \frac{hr}{60\,mi} \times 3.9\,s = \frac{60.1\,ft}{s} \times 3.9\,s = 234\,ft$$

At 53 mph:

$$\frac{53\,mi}{hr} \times \frac{88\,ft}{s} \frac{hr}{60\,mi} \times 4.9\,s = \frac{77.7\,ft}{s} \times 4.9\,s = 381\,ft$$

At 66 mph:

$$\frac{66\,mi}{hr} \times \frac{88\,ft}{s} \frac{hr}{60\,mi} \times 5.8\,s = \frac{96.8\,ft}{s} \times 5.8\,s = 561\,ft$$

f. What is the stopping distance at the above 4 speeds:

Recall that the otal stopping distance = reaction distance + braking distance

$$= v\,(1.5\,s) + \frac{v^2}{2a}$$

At 28mph:

$$\text{Total stopping distance} = \frac{41.1\,ft}{s} \times 1.5\,s + \frac{(41.1\,ft)^2}{s^2} \frac{s^2}{(2)(17\,ft)} = 111\,ft$$

At 41mph:

$$\text{Total stopping distance} = \frac{60.1\,ft}{s} \times 1.5\,s + \frac{(60.1\,ft)^2}{s^2} \frac{s^2}{(2)(17\,ft)} = 197\,ft$$

At 53mph:

$$\text{Total stopping distance} = \frac{77.7\,ft}{s} \times 1.5\,s + \frac{(77.7\,ft)^2}{s^2} \frac{s^2}{(2)(17\,ft)} = 294\,ft$$

At 66mph:

$$\text{Total stopping distance} = \frac{96.8\,ft}{s} \times 1.5\,s + \frac{(96.8\,ft)^2}{s^2} \frac{s^2}{(2)(17\,ft)} = 373\,ft$$

g. Calculate the difference between the constant speed distance and the total stopping distance determined in e and f above.

At 28mph:
Constant speed distance - Total stopping distance
=127 ft - 111 ft = 16 ft

At 41mph:
Constant speed distance - Total stopping distance
=235 ft - 197 ft = 38 ft

At 53mph:
Constant speed distance - Total stopping distance
=381 ft - 294 ft = 87 ft

At 66mph:
Constant speed distance - Total stopping distance
=561 ft - 373 ft = 188 ft

h. Determine the time it takes to travel the distance determined in g. This is the amount of time you have to decide to brake so that you can stop before the light.

Time to travel constant speed distance - total stopping distance
At 28mph:
$\frac{16 \text{ ft s}}{41.1 \text{ ft}} = 0.4 \text{ s}$

So at 28mph, you have 0.4 s of safety margin to decide to brake.

At 41mph:
$\frac{38 \text{ ft s}}{60.1 \text{ ft}} = 0.6 \text{ s}$

So at 41mph, you have 0.6 s of safety margin to decide to brake.

At 53mph:
$\frac{87 \text{ ft s}}{77.7 \text{ ft}} = 1.1 \text{ s}$

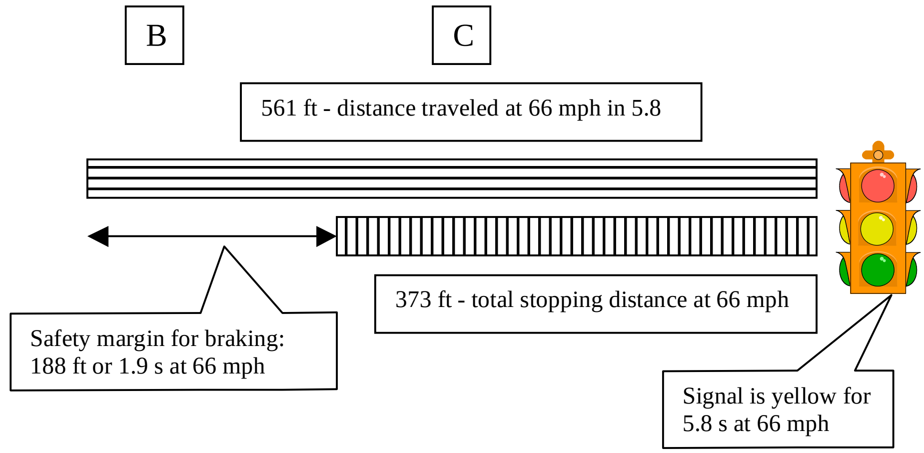
So at 53mph, you have 1.1 s of safety margin to decide to brake.

At 66mph:

$$\frac{188 \text{ ft } s}{96.8 \text{ ft}} = 1.9 \text{ s}$$

So at 66mph, you have 1.9 s of safety margin to decide to brake.

i. Draw a diagram showing the distance traveled at 66 mph, the total stopping distance at 66 mph, and the safety margin in distance and time you have to decide to brake.



At point A, you cannot make it through the light traveling at 66 mph so you should brake and stop. You should know from experience that your car can easily and safely brake at this distance from the light.

At point B, you can either drive at the same speed or brake. You will pass the light before it turns red if you continue traveling at the speed limit. You also have enough time to safely brake if you decide to stop before the light. So at this critical distance from the light, either decision will be a safe one.

At point C, you do not have enough distance to brake before the light. You should know from experience that your car cannot safely brake at this distance from the light. You should continue traveling at the same speed - you will easily pass the light before it turns red.

From the San Diego Union Tribune 10/6/01, page B1.

Headline: "Davis signs bill on red-light cameras; yellow to be timed."

"Gov. Gray Davis signed a bill yesterday requiring that traffic lights with cameras show the yellow caution light for a reasonable amount of time before taking photos of red-light violators.

The amount of time for triggering the automatic cameras would be determined by the California Department of Transportation,

There have been allegations that some traffic lights in San Diego and other areas were set to switch too swiftly from green to red.

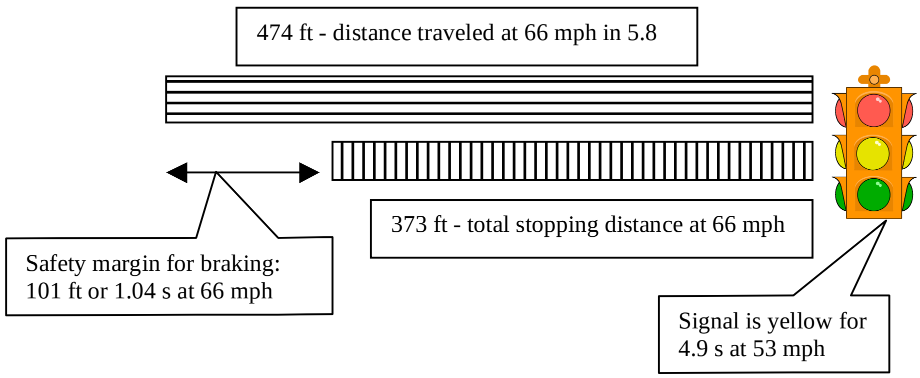
SB 667 by Sen. Steve Peace requires that traffic lights with cameras comply with the Caltrans traffic manual for minimum yellow-light intervals.

The minimum depends on the speed limit. For example, at 25 mph the yellow light must be on for at least 3 seconds. At 45 mph, the interval must be at least 4.3 seconds."

j. Redo the diagram of part i assuming that you are speeding: you are traveling at 66 mph in a zone where the speed limit is 53 mph. At 53 mph, the signal is yellow for 4.9 s.

$$\text{Distance} = 66\text{mph} \times \frac{88\text{ft/s}}{60\text{ mph}} \times 4.9\text{ s} = 474\text{ ft}; \quad 66\text{ mph} \times \frac{88\text{ft/s}}{60\text{ mph}} = 96.8\text{ ft/s}$$

Safety margin time = (474ft - 373ft)/(96.8ft/s) = 1.04 s

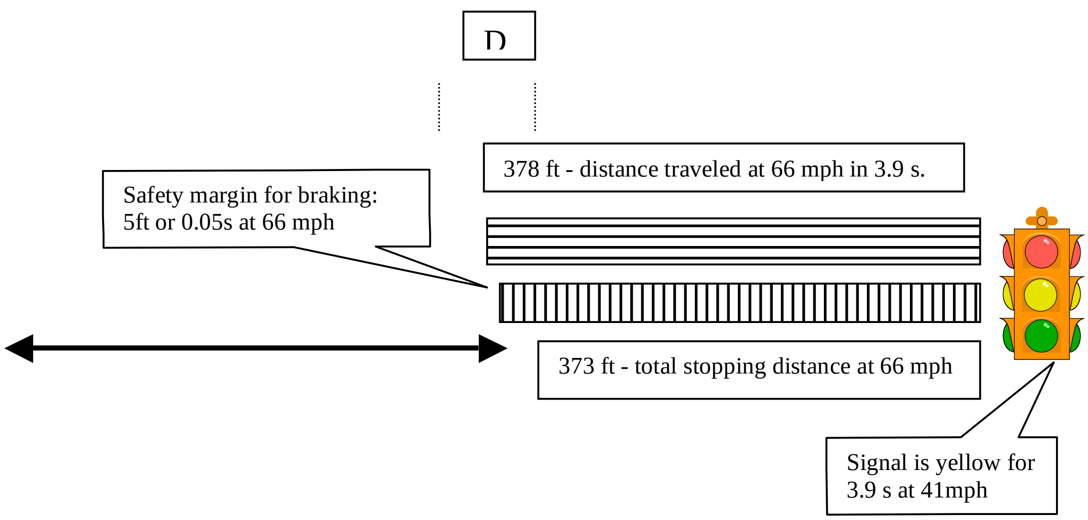


k. Redo the diagram of part i assuming that you are speeding: you are traveling at 66mph in a zone where the speed limit is 41 mph. At 41 mph, the signal is yellow for 3.9 s. What do these results indicate about the dangers of speeding.

$$\text{Distance} = 66\text{mph} \times \frac{88\text{ft/s}}{60\text{ mph}} \times 3.9\text{ s} = 378\text{ ft}; \quad 66\text{ mph} \times \frac{88\text{ft/s}}{60\text{ mph}} = 96.8\text{ ft/s}$$

Safety margin time = (378ft - 373ft)/(96.8ft/s) = 0.05 s

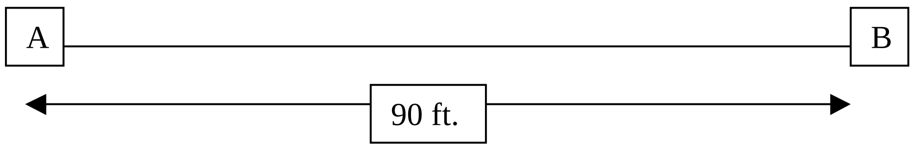
If the car is in the hatched in area shown in the diagram below, it must continue through the light. If it is in the area shown by the arrow, it must stop in order to not run the red light. There is no area B as shown in part i of this problem since the safety margin for making the correct decision is now only 5 ft or 0.05 s when traveling at 66 mph. Near the transition region D, the driver must make an immediate correct decision as to whether or not they should hit the brakes or maintain their speed.



6. A general rule of thumb taught to drivers is to leave one car length of distance between cars for every 10 mph. Assume that a typical car length is 15 feet or 3 meters. Does this make sense? Why or why not? Consider 3 different cases.

a. You are traveling at 60 mph and suddenly the traffic in front of you slows to 50 mph.

The situation is as shown below:



First calculate how long it takes car A to brake from 60 mph to 50 mph.
It takes a driver about 1.5 s to react, so car A travels the following distance:

$x = vt = \frac{88\ \text{ft}}{\text{s}} \times 1.5\ \text{s} = 132\ \text{ft.}$

Next determine the time it takes a car to slow down from 60 mph to 50 mph at a constant rate of deceleration of 17 ft/s^2. Previously, we found that 50 mph = 73.3 ft/s.

Since $a = \frac{v_f - v_I}{t}$, then $t = \frac{v_f - v_I}{a} = \frac{(73.3\ \text{ft} - 88\ \text{ft})\ \text{s}^2}{\text{s} \times (-17\ \text{ft})} = 0.86\ \text{s}$

The average speed of car A during this time is $\frac{v_f + v_I}{2} = \frac{81\ \text{ft.}}{\text{s}}$

So car A travels $x = vt = \frac{81\ \text{ft}}{\text{s}} \times 0.86\ \text{s} = 70\ \text{ft}$ as it decelerates to 50 mph.

So the total time it takes car A to slow down to 50 mph = 1.5 s + 0.86 s = 2.36 s.
The total distance it travels is 132 ft + 70 ft = 202 ft.

In this time interval, car B will travel $x = vt = \frac{73.3\ \text{ft}}{\text{s}} \times 2.36\ \text{s} = 182\ \text{ft.}$

When both cars are traveling at 50 mph, car A will have traveled (202 ft – 182 ft) = 20 feet more than car B. Since this distance is less than 90 ft, car A will not come too close to car B – car A will end up 70 feet behind car B.

b. You are traveling at 60 mph and suddenly the traffic in front of you slows to 40 mph.

First calculate how long it takes car A to brake from 60 mph to 40 mph.
It takes a driver about 1.5 s to react, so car A travels the following distance:
$x = vt = \frac{88\ ft}{s} \times 1.5\ s = 132\ ft.$

Next determine the time it takes a car to slow down from 60 mph to 40 mph at a constant rate of deceleration of 17 ft/s^2. Previously, we found that 40 mph = 49 ft/s.

Since $a = \frac{v_f - v_I}{t}$, then $t = \frac{v_f - v_I}{a} = \frac{(49\ ft - 88\ ft)\ s^2}{s \times (-17\ ft)} = 2.3\ s$

The average speed of car A during this time is $\frac{v_f + v_I}{2} = \frac{68.5\ ft.}{s}$

So car A travels $x = vt = \frac{68.5\ ft}{s} \times 2.2\ s = 151\ ft$ as it decelerates to 40 mph.

So the total time it takes car A to slow down to 50 mph = 1.5 s + 2.3 s = 3.8 s. The total distance it travels is 132 ft + 151 ft = 283 ft.

In this time interval, car B will travel $x = vt = \frac{49\ ft}{s} \times 3.8\ s = 186\ ft.$

When both cars are traveling at 40 mph, car A will have traveled (283 ft – 186 ft) = 97 feet more than car B. Since this distance is more than 90 ft, car A will hit car B.

c. You are travelling at 60 mph and all of a sudden you become aware that the traffic is stopped in front of you when the traffic is only 6 car lengths in front of you.

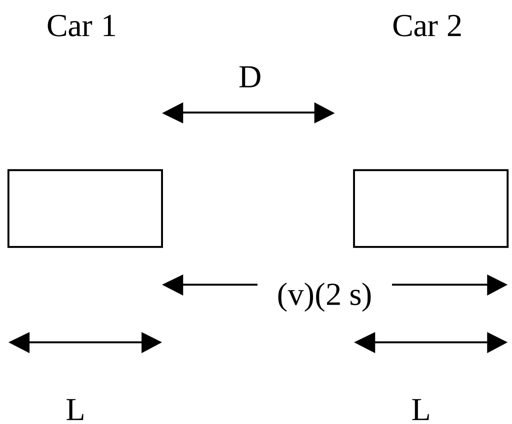
The total stopping distance at 60 mph is 360 ft (see problem 5f), much more than the distance of 6 car lengths of 90 ft. So you will hit the car in front of you. It takes you 1.5 s to react to the situation. During this time, your car travels 132 ft, so you will still be traveling at 60 mph when you hit the car in front of you – which will probably result in serious injury or death.

In conclusion, as long as you keep your relative speed to within about 15 mph of the traffic in front of you, you will be able to react in time.

7. Another way to judge appropriate spacing between cars when driving is to consider what is called "headway." Headway is defined as the elapsed time between the front of the lead vehicle passing a point on the roadway and the front of the following vehicle passing

the same point. Most driving manuals recommend a headway of at least 2 s. How does a headway of 2-s compare to the rule of thumb that you should leave 1 car length between the front of your car and the back of the car in front of you for every 10-mph of speed? Hint 1: Assume that each car length is 14.7 ft long. Recall that 10mph = 14.7 ft/s.
Hint 2: Assume two cars are moving at the same constant speed, one behind the other. Call the speed v. At time t = 0 the front of the lead vehicle passes a given point on the highway. Two seconds later the front of the second vehicle passes that same point. The total distance between the front of the two vehicles is therefore (v)(2 s).

Assume two cars are moving at the same constant speed, one behind the other. Call the speed v. At time t = 0 the front of the lead vehicle passes a given point on the highway. Two seconds later the front of the second vehicle passes that same point. The total distance between the front of the two vehicles is therefore (v)(2 s). Thus the distance D between the back of the first car and the front of the second is (v) (2 s) – L, where L is the length of the car.



The lengths of cars vary, typically ranging around 14-15 ft. For simplicity, take L = 14.7 ft. This gives us

$$D = (v)(2\ s) - L = (v)(2\ s) - 14.7ft$$

For v = 10mph, v = 14.7 ft/s
so

$$D = \frac{(14.7\ ft)(2\ s)}{s} - 14.7\ ft = 14.7\ ft = 1 \text{ car length}$$

For v = 60mph, v = 88 ft/s
so

$$D = \frac{(88ft)\,(2\,s)}{s} - 14.7\text{ ft} = 161.3\text{ ft}$$

$$\frac{161.3\text{ ft} \times 1\text{ car length}}{14.7\text{ ft}} = 11\text{ car lengths}$$

Thus, the distance between cars varies from 1 car length for every 10mph (at low speeds) to a bit less than 2 car lengths for every 10mph (at high speeds), which is in accord with the rough rule of thumb but is better because it allows for greater spacing at higher speeds.

Not only is a headway of 2 seconds a safer rule of thumb, you can more readily measure headway on the road. Estimating the distance between vehicles is more difficult than noting the time between cars passing the same point on the road.

Excerpt from Traffic Safety and the Driver (p. 314-316): "... drivers who are following other vehicles do so with an average headway of 1.32 seconds; that is, the average headway is considerably shorter than the recommended minimum. ... Why do drivers choose to follow so closely? It seems to me that it becomes largely a driving habit, rather than reasoned conscious behavior. ... Following at a headway of 2.0 seconds instead of 0.5 seconds means you will arrive 1.5 seconds later, assuming that no vehicles cut in front of you. ... Even if a few vehicles do cut into the gap in front ... this adds only about 2 seconds per such incident to the overall trip time. Drivers probably object to other vehicles cutting in front of them not because it delays them a couple of seconds, but because it is interpreted as some sort of personal affront, an assault on manhood or womanhood. If detached rationality cannot dispel such feelings, comfort might be sought in the confident expectation that the offending driver is likely to be experiencing more than the average crash rate of one per 10 years. Let such drivers have their fun - they are paying a high price for it; recapture your two seconds by walking faster to your vehicle."

8. You are driving at night to a friend's house. You make a sharp right hand turn onto another street and suddenly, 75 feet in front of you, you see someone crossing the street. Based on the results of problem 5, what is the maximum speed you could be traveling at and not hit the person.

Based on problem 5, the maximum speed is about 20 mph, since at 20 mph, the total stopping distance is about 70 ft.

How fast should you be driving when making a turn onto a street?

You speed should be such that your stopping distance is less than the distance your lights are illuminating in front of you. When making a turn, that distance may be as little as 25 feet, so you should not make turns at night at speeds higher than about 10 mph.

9. Headlights illuminate the road up to 160 feet in front of you. If you are on a road with stop signs, what is the fastest speed you can drive and still stop safely at night?

The total stopping distance must be less than or equal to 160 feet we will assume it is equal to 160 ft so that your car will come to a complete stop right at the stop sign. If you are initially travelling at a speed v, then the reaction distance is vt and the braking distance is $v^2/(2a)$.

So the total stopping distance (d_{total}) is:

$$d_{total} = vt + \frac{v^2}{2a}.$$

d_{total} = 160 ft, the braking deceleration $a = \frac{17 \text{ ft}}{s^2}$, and the reaction time t = 1.5 s.

The equation that must be solved is a quadratic equation:

$$\frac{v^2}{2a} + vt - d_{total} = 0 \quad \text{or} \quad \frac{0.029\ s^2\ v^2}{ft} + 1.5s\ v - 160ft = 0$$

Using the quadratic formula

$$v = \frac{-1.5\ s\ +/-\ sqr\ \{1.5^2 s^2 - 4 * .029 s^2/ft * (-160ft)\}}{2 * \ 0.029\ s^2/ft}$$

Only the + sign yields a physically meaningful solution, so:

$$v = \frac{52.8 \text{ ft}}{s} \text{ or 36 mph.}$$

Perhaps this is why the speed limit on country roads, where you may encounter a stop sign without warning, is often 35 mph.
If you were drunk and traveling at this speed, your reaction time would double to 3 s. So your reaction distance would be:

$$\text{Reaction distance} = vt = \frac{52.7 \text{ ft}}{s} \times 3\text{ s} = 158.1 \text{ ft.}$$

$$\text{Your braking distance is still } \frac{v^2}{2a} = \frac{52.7^2\ ft^2\ s^2}{s^2\ 2 \times 17\ ft} = 81.7 \text{ ft}$$

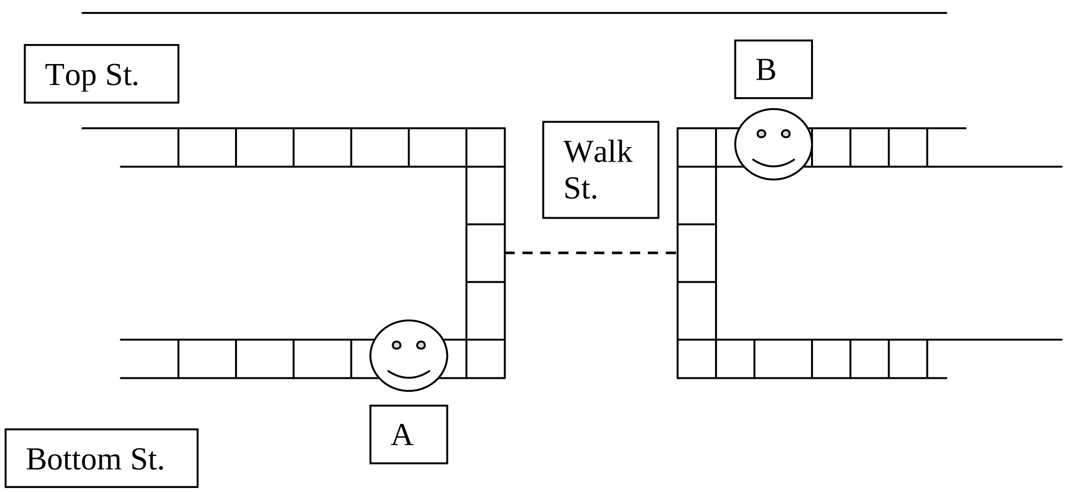
So the total braking distance = 151.8 ft + 81.7 ft = 239.8 ft. This distance is greater than you can see using your lights (160 ft), so you would not be able to stop before the stop sign. You will be traveling through the intersection, possibly causing a crash.

9A. You are traveling on a freeway with a small amount of traffic. After about a hour of driving, you notice that there appear to be no cars traveling at your speed - all cars seem to be either passing you or you are passing them. Why is this?

The reason is that cars traveling at the same speed as you never get closer to you or farther from you - they stay the same distance from you. So you will never see the cars traveling at your speed.

9B. Smiley face wants to walk from the sidewalk at point A to the sidewalk at point B at night. She needs to cross the street. Where is the safest place for Smiley to cross and why?

Smiley should cross at the dashed line since it gives her the most time to react to a car entering Walk St. from either Top St. or Bottom St.



10. A fire engine is traveling at 25 m/s directly towards a bus station on its way to a fire. At its closest approach it passes right next to the station. Starting at 400 m before the station, it sends out a very short blast of sound every 100 m. It stops sending these messages when it is 400 m past the station. Sound travels at 330 m/s. If you are standing at the bus station, determine the time interval between successive blasts of sound. Calculate and compare (using a table and a chart) how the time intervals change when the fire engine is approaching you versus when it is moving away from you.

At time t=0, the first sound is sent out. At this point, the fire engine is 400 meters away, so the sound takes $\Delta t = d/v = 400\ s/330 = 1.2$ s. So the first sound appears at the station at t = 1.2 s.

After the fire engine has traveled 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 4 s. The sound must now travel 300 m, which takes $\Delta t = d/v = 300\ s/330 = 0.9$ s. So this sound appears at the station at t = 4.9 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 8 s. The sound must now travel 200 m, which takes $\Delta t = d/v = 200\ s/330 = 0.6$ s. So this sound appears at the station at t = 8.6 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 12 s. The sound must now travel 100 m, which takes $\Delta t = d/v = 100\ s/330= 0.3$ s. So this sound appears at the station at t = 12.3 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. It is now at the station. So this sound is sent out at t = 16 s. The sound must now travel 0 m, which takes $\Delta t = d/v = 0\ s/330 = 0$ s. So this sound appears at the station at t = 16 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. It is now 100 m past the station. So this sound is sent out at t = 20 s. The sound must now travel 100 m, which takes $\Delta t = d/v = 100\ s/330 = 0.3$s. So this sound appears at the station at t = 20.3 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 24 s. The sound must now travel 200 m, which takes $\Delta t = d/v = 200\ s/330 = 0.6$ s. So this sound appears at the station at t = 24.6 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 28 s. The sound must now travel 300 m, which takes $\Delta t = d/v = 300\ s/330 = 0.9$ s. So this sound appears at the station at t = 28.9 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 32 s. The sound must now travel 400 m, which takes $\Delta t = d/v = 400\ s/330 = 1.2$ s. So this sound appears at the station at t = 33.2 s.

Make a table summarizing the data.

Distance of fire engine from station (m)	Time signal arrived at station (s)	Time between successive signals (s)
-400	1.2	
-300	4.9	3.7
-200	8.6	3.7
-100	12.3	3.7
0	16.0	3.7
100	20.3	4.3
200	24.6	4.3
300	28.9	4.3
400	33.2	4.3

So the time between successive blasts of sounds is less as the fire engine is approaching the station compared with the time between successive blasts of sound as the fire engine travels away from the station. This is the origin of the Doppler effect.

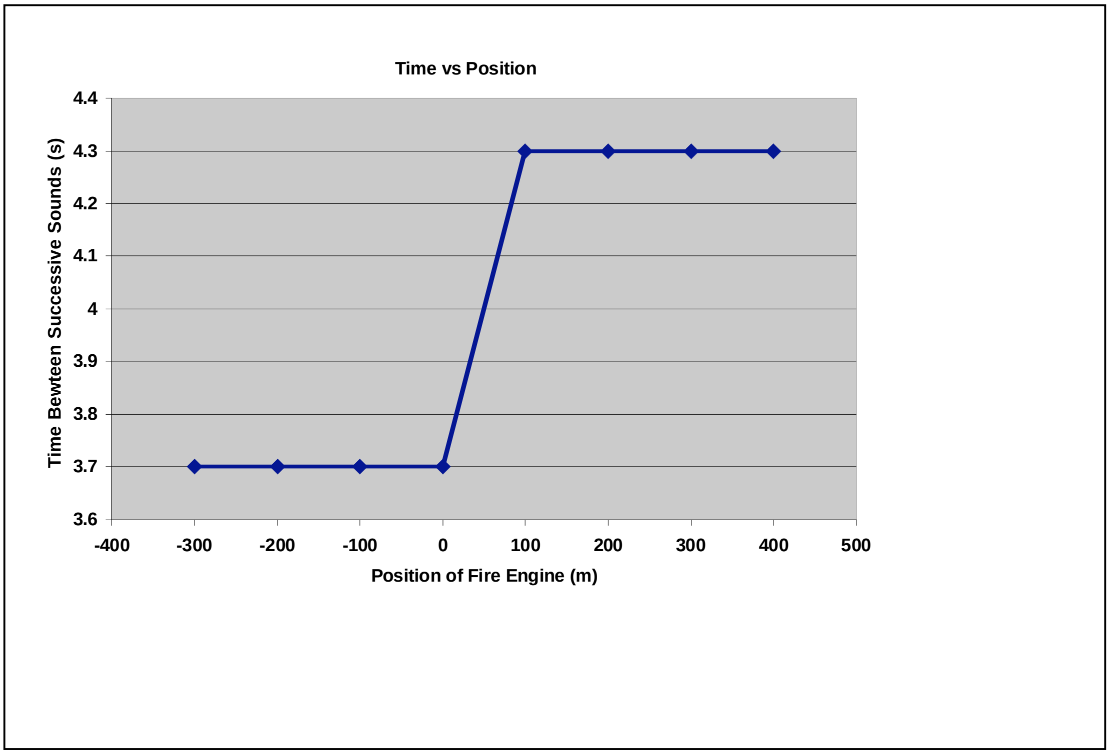
Note that there is a difference between the fire engine approaching the station and receding from the station.

As the fire engine is approaching the station:

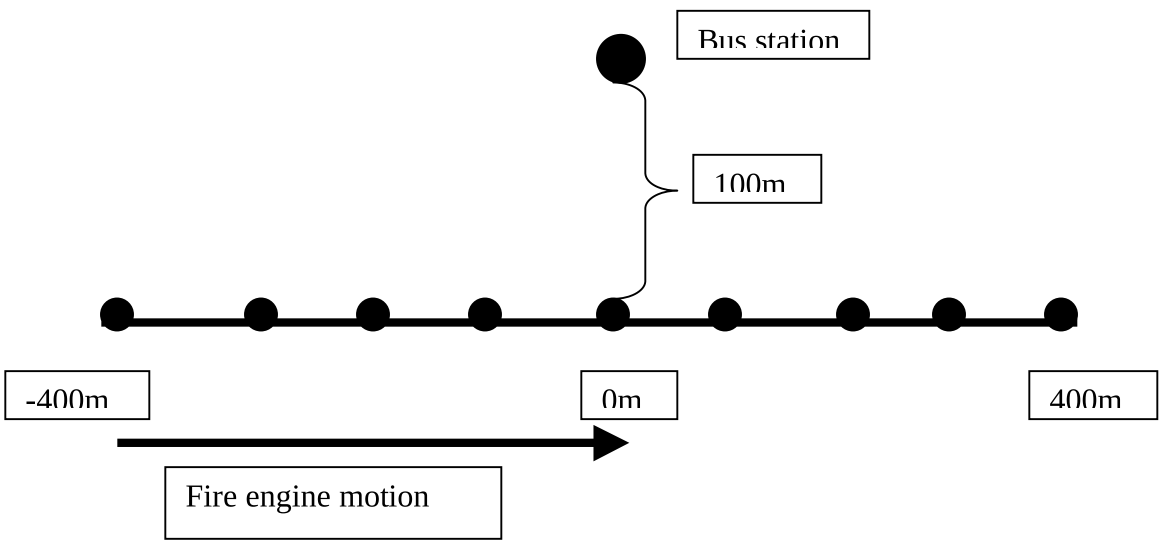
- it sends out a signal
- it travels toward the station
- it sends out the next signal when it is closer to the station

As the fire engine is receding from the station:

- it sends out a signal
- it travels away from the station
- it sends out the next signal when it is farther from the station



10A. A fire engine is traveling at 25 m/s on its way to a fire. At its closest approach it passes 100m from a bus station. Starting at 400 m before the station, it sends out a very short blast of sound every 100 m. It stops sending these messages when it is 400 m past the station. Sound travels at 330 m/s. If you are standing at the bus station, determine the time interval between successive blasts of sound. Calculate and compare (using a table and a chart) how the time intervals change when the fire engine is approaching you versus when it is moving away from you.



At time t=0, the first sound is sent out. At this point, the fire engine is $(400^2 + 100^2)^{0.5}$ m = 412.3 m away, so the sound takes $\Delta t = d/v$ = 412.3s/330 = 1.249 s. So the first sound appears at the station at t = 1.249 s.

After the fire engine has traveled 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 4 s. The sound must now travel $(300^2 + 100^2)^{0.5}$ m = 316.2 m, which takes $\Delta t = d/v$ = 316.2 s/330 = 0.958 s. So this sound appears at the station at t = 4.958 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 8 s. The sound must now travel $(200^2 + 100^2)^{0.5}$ m = 223.6 m, which takes $\Delta t = d/v$ = 223.6 s/330 = 0.678 s. So this sound appears at the station at t = 8.678 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 12 s. The sound must now travel $(100^2 + 100^2)^{0.5}$ m = 141.4 m, which takes $\Delta t = d/v$ = 141.4 s/330= 0.428 s. So this sound appears at the station at t = 12.428 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. It is now at its closest approach to the bus station. So this sound is sent out at t = 16 s. The sound must now travel 100 m, which takes $\Delta t = d/v$ = 100 s/330 = 0.303 s. So this sound appears at the station at t = 16.303 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 20 s. The sound must now travel $(100^2 + 100^2)^{0.5}$ m = 141.4 m, which takes $\Delta t = d/v$ = 141.4 s/330= 0.428 s. So this sound appears at the station at t = 20.428 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 24 s. The sound must now travel $(200^2 + 100^2)^{0.5}$ m = 223.6 m, which takes $\Delta t = d/v$ = 223.6 s/330 = 0.678 s. So this sound appears at the station at t = 24.678 s.

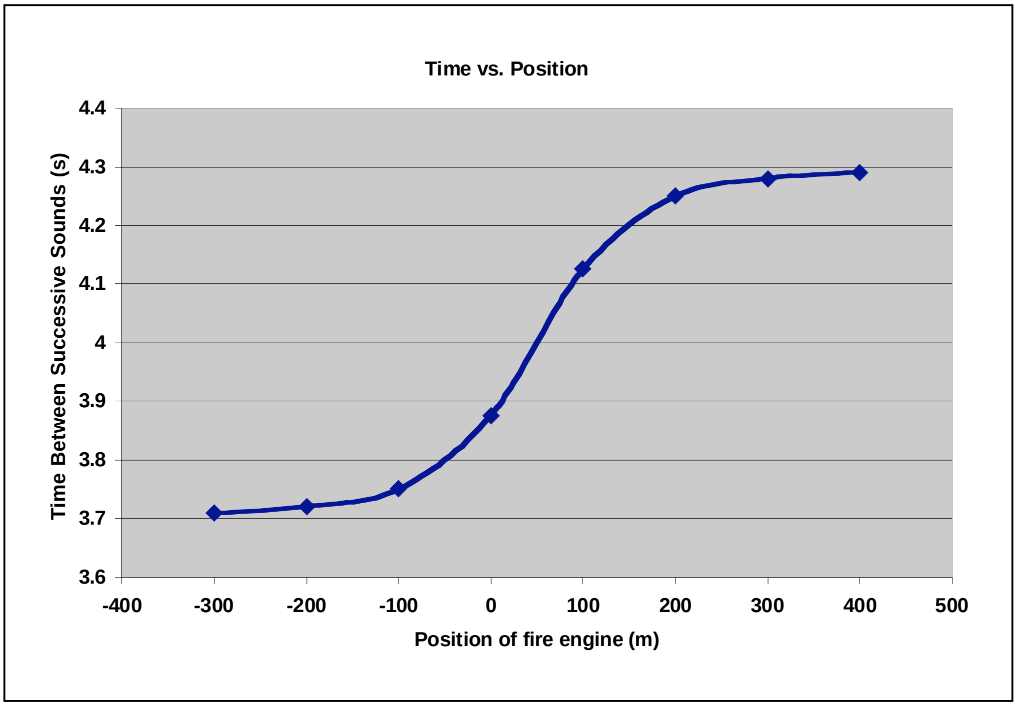
After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 28 s. The sound must now travel $(300^2 + 100^2)^{0.5}$ m = 316.2 m, which takes $\Delta t = d/v$ = 316.2 s/330 = 0.958 s. So this sound appears at the station at t = 28.958 s.

After the fire engine has traveled another 100 m (which takes 4 s at 25 m/s), the fire engine sends out another sound. So this sound is sent out at t = 32 s. The sound must now travel $(400^2 + 100^2)^{0.5}$ m = 412.3 m, so the sound takes $\Delta t = d/v$ = 412.3s/330 = 1.249 s. So this sound appears at the station at t = 33.249 s.

Make a table summarizing the data.

Distance of fire engine from station (m)	Time signal arrived at station (s)	Time between successive signals (s)
-400	1.249	
-300	4.958	3.709
-200	8.678	3.720
-100	12.428	3.750
0	16.303	3.875
100	20.428	4.125
200	24.678	4.250
300	28.958	4.280
400	33.249	4.291

So the time between successive blasts of sounds slowly increases as the fire engine is approaching the bus station, then increases rapidly as it passes its closest approach to the bus station, then slowly increases again as it travels away from the bus station. This is the origin of the Doppler effect.



11. Jack and Jill each drive their vehicles 10,000 miles per year. Jack's vehicle has a fuel economy of 10 miles per gallon, Jill's 30 miles per gallon.

a. How much fuel does each of them use in a year?

Jack: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{10 \text{ mi}} = \frac{1000 \text{ gallons}}{\text{year}}$

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{30 \text{ mi}} = \frac{333 \text{ gallons}}{\text{year}}$

b. How much fuel does the Jack and Jill household use in a year?

The Jack and Jill household uses 1333 gallons per year.

c. How far do they travel in a year?

The Jack and Jill household travels a total of 20,000 miles.

d. What is their average household fuel economy? Is it the average of Jack's fuel economy and Jill's fuel economy?

The Jack and Jill household average fuel economy is given by the total miles driven divided by the total fuel used, so

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1333 \text{ gallons}} = \frac{15 \text{ miles}}{\text{gallon}}$.

Note that this is NOT the average of Jack's fuel economy and Jill's fuel economy, which would be 20 miles per gallon.

e. What would their average household fuel economy be if Jill's vehicle got 100 miles per gallon?

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{100 \text{ mi}} = \frac{100 \text{ gallons}}{\text{year}}$

So the total household would use 1100 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1100 \text{ gallons}} = \frac{18.2 \text{ miles}}{\text{gallon}}$.

f. What would their average household fuel economy be if Jill's vehicle got 1000 miles per gallon?

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{1000 \text{ mi}} = \frac{10 \text{ gallons}}{\text{year}}$

So the total household would use 1010 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1010 \text{ gallons}} = \frac{19.8 \text{ miles}}{\text{gallon}}$.

g. What would their average household fuel economy be if Jill's vehicle got 10,000 miles per gallon?

Jill: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{10{,}000 \text{ mi}} = \frac{1 \text{ gallon}}{\text{year}}$

So the total household would use 1001 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{1001 \text{ gallons}} = \frac{19.98 \text{ miles}}{\text{gallon}}$.

h. What would their average household fuel economy be if Jack's vehicle got 30 miles per gallon, the same as Jill's original vehicle?

Jack: $\frac{10{,}000 \text{ mi}}{\text{year}} \times \frac{1 \text{ gallon}}{30 \text{ mi}} = \frac{333 \text{ gallon}}{\text{year}}$

So the total household would use 666 gallons, therefore:

Household average fuel economy = $\frac{20{,}000 \text{ miles}}{666 \text{ gallons}} = \frac{30 \text{ miles}}{\text{gallon}}$.

i. If you were in charge of making policy to reduce fuel consumption, what would you do?

It is far better to try to improve the fuel economy of the worst vehicles than the best vehicles. Therefore, you might try to make a minimum fuel economy standard or, as Congress has done, mandate a required average fuel economy for all vehicles sold by a car manufacturer.

12. Consider the following distribution of dots on the line below. Let's call the dots "galaxies" and let's call the line "the universe." Suppose that adjacent galaxies are all located a distance of L apart from each other in the universe. At a time T later, the universe has expanded a factor of two so that now all of the adjacent galaxies are a distance of 2L apart.

a. Suppose you are living in galaxy A. How fast does it appear that galaxies B, C, and D are receding from you?

From the perspective of galaxy A, galaxy B traveled a distance of L in time T so its speed of recession is L/T. From the perspective of galaxy A, galaxy C traveled a distance of 2L in time T so its speed of recession is 2L/T. From the perspective of galaxy A, galaxy D traveled a distance of 3L in time T so its speed of recession is 3L/T.

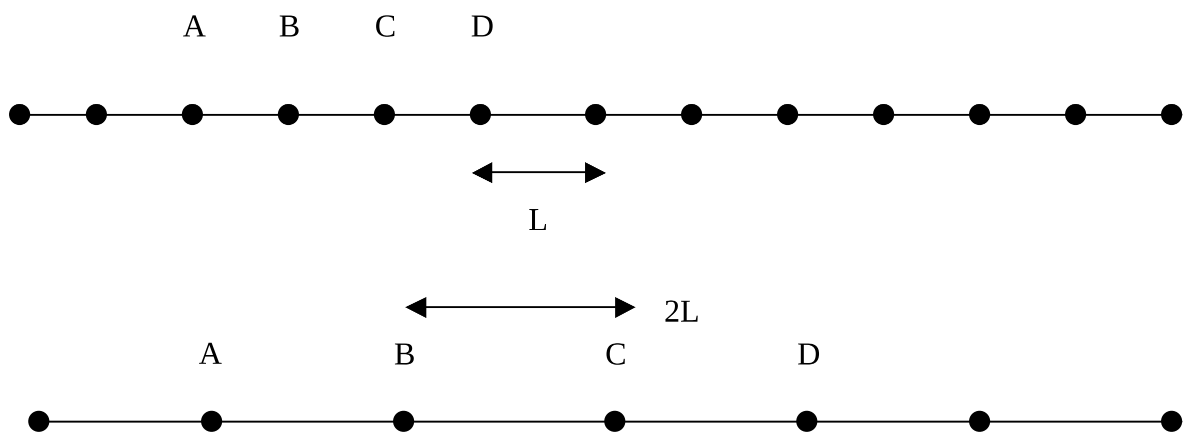
b. Is there a correlation between the distance the galaxy is located from you and the speed with which it is receding from you. What is that relationship?

The further away the galaxy is from you, the faster it appears to be moving. The relationship (known as the Hubble Constant in astronomy) is that the speed of recession is L/T for every distance L the galaxy is located from you. These data are summarized in the table below.

Galaxy	Original distance of galaxy from galaxy A	Distance traveled by galaxy in time T as observed by galaxy A	Speed of recession of galaxy as observed by galaxy A	Speed of recession of galaxy as observed by galaxy A divided by original distance of galaxy from galaxy A
B	L	L	L/T	1/T {or (L/T)/L}
C	2L	2L	2L/T	1/T {or (2L/T)/(2L)}
D	3L	3L	3L/T	1/T {or (3L/T)/(3L)}

c. Do all galaxies see the same thing happening?

Yes. To a person on any galaxy, it appears that all the other galaxies are moving away from them and the speed of recession is proportional to the distance from your galaxy.



13. From the Associated Press dated 11/4/02: “Long Beach – Nearly 200 cars and big-rig trucks collided in two incidents on the fogbound Long Beach Freeway early yesterday, injuring dozens of people, nine critically, and closing the highway for hours… CHP officer Joseph Pace … said visibility was down to about 50 feet in heavy fog when the chain reaction crashes began just before 7 a.m … Some motorists estimated cars were moving at 25 to 35 mph.”

From the Associated Press dated 11/5/02: “Los Angeles – The chain-reaction crashes that piled up nearly 200 cars on the Long Beach Freeway likely could have been avoided if drivers had simply slowed down when they hit foggy conditions, California Highway Patrol officers said yesterday. The crashes, which left a five-mile section of the freeway looking like an auto junkyard, shut down the highway for 11 hours Sunday. Eight people suffered critical or serious injuries in the accidents, which took place within minutes. Motorists reported driving into fog so thick it reminded some of being on an airliner as it travels into the clouds. “In that weather condition, we’re sure if drivers had drastically reduced their speeds, this could have been avoided,” said California Highway Patrol Officer Luis Mendoza.”

a. What is the total breaking distance for a car traveling at 25 mph?
b. What is the total breaking distance for a car traveling at 35 mph?
c. How do these total breaking distances compare to the 50 foot visibility of that day? Why did the accidents occur?
d. What maximum speed should the cars have been traveling at if the visibility was only 50 feet.

a. $\frac{25\text{ mi}}{\text{hr}} \times \frac{88\text{ft hr}}{\text{s } 60\text{ mi}} = \frac{36.7\text{ ft}}{\text{s}}$

Reaction distance $= \frac{36.7\text{ ft}}{\text{s}} \times 1.5\text{ s} = 55\text{ ft}$

Braking distance $= \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$

$= \frac{36.7^2\text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 39.6\text{ ft}$

So stopping distance = reaction distance + braking distance = 55 ft + 39.6 ft = 94.6 ft.

b. a. $\frac{35 \text{ mi}}{\text{hr}} \times \frac{88\text{ft} \;\; \text{hr}}{\text{s} \;\; 60 \text{ mi}} = \frac{51.3 \text{ ft}}{\text{s}}$

Reaction distance $= \frac{51.3 \text{ ft}}{\text{s}} \times 1.5 \text{ s} = 77 \text{ ft}$

Braking distance $= \frac{(\text{initial speed})^2}{2 \times \text{deceleration}}$

$$= \frac{51.3^2 \text{ ft}^2 \times \text{s}^2}{\text{s}^2 \times 2 \times 17\text{ft}} = 77.4 \text{ ft}$$

So stopping distance = reaction distance + braking distance = 77 ft + 77.4 ft = 154.4 ft.

c. The total stopping distances for cars traveling at 25 and 35 miles per hour is 94 and 154 feet respectively, much greater than the visibility of 50 feet. So the cars could not stop in time to avoid a crash.

d. The total stopping distance is equal to 50 ft. If you are initially traveling at a speed v, then the reaction distance is vt and the braking distance is $v^2/(2a)$.

.

So the total stopping distance (d_{total}) is:

$$d_{total} = vt + \frac{v^2}{2a}.$$

$d_{total} = 50$ ft, the braking deceleration $a = \frac{17 \text{ ft}}{\text{s}^2}$, and the reaction time t = 1.5 s.

The equation that must be solved is a quadratic equation:

$$\frac{v^2}{2a} + vt - d_{total} = 0 \text{ or } \frac{0.029 \text{ s}^2 v^2}{\text{ft}} + 1.5\text{s } v - 50\text{ft} = 0$$

Using the quadratic formula

$$v = \frac{-1.5 \text{ s} +/- \text{sqr} \{1.5^2\text{s}^2 - 4 * .029\text{s}^2/\text{ft} * (-50\text{ft})\}}{2 * 0.029 \text{ s}^2/\text{ft}}$$

Only the + sign yields a physically meaningful solution, so:

$v = \frac{23 \text{ ft}}{\text{s}}$ or 16 mph.

So if the visibility was only 50 feet, cars should have been traveling at a maximum speed of 16 mph.

14. From the January/February 2003 issue of the AAA magazine entitled "A Glaring Concern: HID headlights: boon to vehicular safety or blight on the automotive landscape?"

"Nevertheless, the visual improvement high-intensity discharge (HID) lamps provide is dramatic: A motorist using HIDs can see about 330 feet in front of the vehicle, compared with 190 feet with standard halogen lighting – almost a 75 percent improvement."

"One of the problems with halogen lighting is that motorists often 'overdrive' their headlights … they're unable to see people, animals, or objects until it's too late."

"The data are startling: Under perfect conditions (dry pavement, mechanically sound vehicle, antilock brakes, alert and skilled driver) at 45 mph, it takes 170 feet to stop your car; at 50 mph, 205 feet; and at 60 mph, 282 feet. So, somewhere between 45 and 50 mph, you've overdriven halogen headlights."

a. What reaction time/breaking time is being assumed for the "perfect conditions" driver discussed above? Is this a fair discussion?

b. Discuss how HIDs can improve driving safety given your work so far in this unit. Consider the average driver with an average car and average reaction times.

Acknowledgements

I would like to thank Ms. Amy Bering, Research and Evaluation, NHTSA, for supplying information about braking distances and about the effect of alcohol on the reaction time of drivers. I would also like to thank Craig Bohren and Leonard Evans for comments, suggestions, improvements and corrections. I am grateful to Robert T. Johnson, Jr. of the City of Carlsbad transportation division for providing the California traffic manual information regarding the timing of yellow lights.

The Doppler problem was inspired by Craig Bohren's treatment of the Doppler effect in his book, What Light Through Yonder Window Breaks, pp. 156-158. This book, as well as Bohren's other popular book, Clouds in a Glass of Beer, are treasures and I highly recommend them. He also suggested that I include a problem on headway.

Leonard Evans suggested the gas mileage problem to me. His book, Traffic Safety and the Driver is the classic if you want to find out additional information about this topic.

References:

1. Traffic Safety and the Driver, Leonard Evans, Van Nostrand Reinhold, New York, 1991.

2. "Car Collisions, Physics, and the State Highway Patrol," David Larabee, The Physics Teacher Vol. 38, Sept. 2000, pp. 334-336.

3. “Simple Kinematics and Traffic Flow,” C. H. Worner, S. Romero, and A. Romero, The Physics Teacher Vol. 37, Sept. 1999, pp. 354-355.
See also the letter to the editor by Richard Hall in The Physics Teacher Vol. 37, Dec. 1999, p. 518, written in response to the above article.

4. The National Highway Transportation Safety Administration “Safe and Sober” web site:
http://www.nhtsa.dot.gov/people/outreach/safesobr/15qp/web/mpmyouth.html
http://www.nhtsa.dot.gov/people/outreach/safesobr/15qp/web/iddrug.html

5. The Department of Transportation “Don’t drink and drive” web site
http://www.transport.gov.za/projects/arrive/alcohol98.html

6. The National Highway Transportation Safety Administration “Consumer Braking Information” web site:
http://www.nhtsa.dot.gov/cars/testing/brakes/

7. The Physics of Racing, Part 3: Basic Calculations
http://members.home.net/rck/phor/03-Basic-Calcs.html

8. The Physics of Racing, Part 11: Braking
http://members.home.net/rck/phor/11-Braking.html

9. Yellow Traffic Lights
http://www2.msmary.edu/math-cs/nsf/1994/10-1994.html

10. "Traffic lights must meet strict standards," R. John Koshel, Optoelectronics World (A supplement to Laser Focus World magazine), October 2000, pp. S15-S19

Appendix A: Brake Stop Results for Popular Cars

Average Brake Stop Results from 100km/hr (62 mph)
(From: The National Highway Transportation Safety Administration "Consumer Braking Information" web site: http://www.nhtsa.dot.gov/cars/testing/brakes/)

Vehicle	Dry surface stopping distance (ft)	Dry surface deceleration rate (ft/s^2)	Wet surface stopping distance (ft)	Wet surface deceleration rate (ft/s^2)
Pontiac Grand Am SE	147.9	26.2	190.1	20.4
Ford Expedition	170.4	22.7	198.9	19.5
Toyota Camry	159.7	24.2	175.7	22.0
Chevy Malibu LS	141.3	27.4	150.3	25.8
Dodge Caravan SE	159.8	24.2	166.3	23.3
Chevrolet Astro	170.2	22.7	174.9	22.1
Dodge Ram 1500 4x4	199.2	19.4	209.6	18.5

Road friction measurements were measured to be 0.90 for dry pavement and 0.85 for wet pavement. Water depth was generally below 3mm (1/8 inch). Some hydroplaning was experienced when there was standing water only 1/4 inch deep that had collected in minor depressions on the test course.

Activity Suggestions:
1. Redo the investigations in this unit using the above values for wet and dry surface deceleration.
2. Determine the deceleration rate for the car in which the student typically drives. Use the web or past issues of car magazines to find stopping distances or deceleration rates.
3. Calculate the coefficient of friction using these data.

Appendix B: Effect of alcohol on reaction times

Alcohol is water-soluble and is readily absorbed in the blood. More blood is supplied to the brain that to other organs, with the result that alcohol impairs your brain function within minutes. At a blood alcohol content (BAC) of 0.08 gm/100ml, the reaction time of the average driver doubles from 1.5 s to 3.0 s. Muscle coordination also diminishes and a driver is more likely to respond incorrectly to stimuli. A 1997 New England Journal of Medicine cited a study that found that talking on a cell phone quadruples a driver’s risk of collision, roughly the same as being drunk. Studies have shown that BAC levels as low as 0.04 gm/100 ml can affect reaction times. Simple reaction times (where the subject attempts to detect a stimulus and respond as quickly as possible) appear to be less affected by lower BACs than do complex reaction times (where the subject must discriminate between stimuli and respond appropriately.) If your BAC is 0.08 gm/100 ml, you are 4 times more likely to crash than if you are sober. At a BAC of 0.12 gm/100 ml, your chances are 15 times more likely and at a BAC of 0.16 gm/100 ml, your chances of crashing are 30 times more than if you are sober.

According to an article in the 1/14/01 issue of Parade Magazine, “Three out of four teens say that they speed when they drive, and about half don’t wear seat belts. Plus, 40% say they’ve ridden with a teen driver who was intoxicated or impaired.”

Appendix C: Traffic Manual Information on Yellow and Red Lights

According to the state of California traffic manual, section 9-04.5:

"The purpose of the yellow signal indication is to warn traffic approaching the signal that the related green movement is ending or that a red indication will be exhibited thereafter and traffic will be required to stop when the red signal is exhibited.

The length of the yellow change interval is dependent upon the speed of approaching traffic. Suggested yellow intervals are shown below:"

Approach Speed (km/hr)	Yellow interval (s)
45 or less	3.1
50	3.3
55	3.5
60	3.7
65	3.9
70	4.2
75	4.4
80	4.7
85	4.9
90	5.1
95	5.3
100	5.5
105	5.8
110	6.0

Section 9-04.6 Red Clearance Intervals

"Generally, red clearance intervals are not required. A red clearance interval may be used following the yellow change interval, at very wide intersections, offset intersections, or at other locations where it is desirable to delay the green interval for opposing traffic. Normally, red clearance intervals range from 0.1 to 2.0 s."

Extra: According to Bob Johnson, traffic engineer for the city of Carlsbad, CA, "We are converting our red and green traffic signal indications to LEDs. The LED uses about 12-14 watts compared to the 150-watt bulbs in the red and green indications. This results in tremendous energy savings."

As an interesting exercise, have your students calculate the expected yearly energy savings from switching from the incandescent bulbs to the LEDs for the green and red signal indications.