

Investigation #5 – Position, speed, and acceleration

1. A car is initially at a position of $x = 0$ ft. It is traveling in the positive direction at a speed of 88 ft/s. What is the car's position after:

- 1 s
- 2 s
- 3 s
- 4 s
- 5 s
- 6 s
- Draw a graphical representation of these results.
- Describe the motion of the car.

Solution:

1. $x_f = x_i + vt$

a. $x = 0 \text{ ft} + vt = (88 \text{ ft/s})(1 \text{ s}) = 88 \text{ ft}$

b. $x = 0 \text{ ft} + vt = (88 \text{ ft/s})(2 \text{ s}) = 176 \text{ ft}$

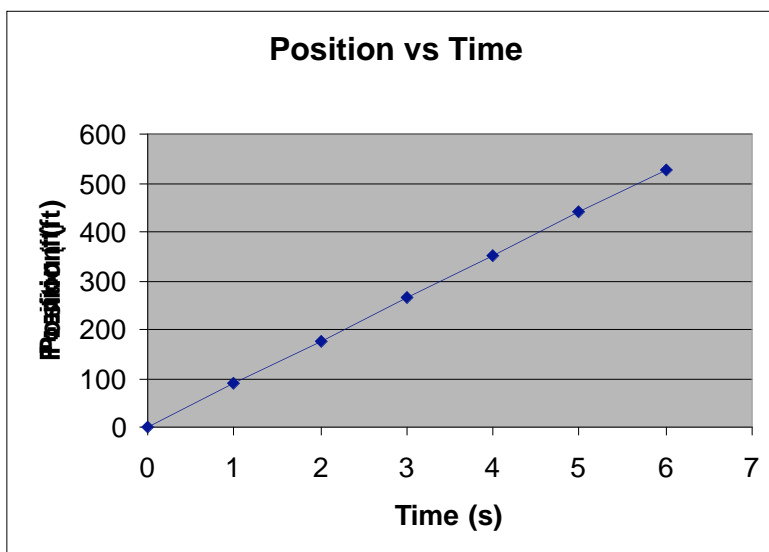
c. $x = 0 \text{ ft} + vt = (88 \text{ ft/s})(3 \text{ s}) = 264 \text{ ft}$

d. $x = 0 \text{ ft} + vt = (88 \text{ ft/s})(4 \text{ s}) = 352 \text{ ft}$

e. $x = 0 \text{ ft} + vt = (88 \text{ ft/s})(5 \text{ s}) = 440 \text{ ft}$

f. $x = 0 \text{ ft} + vt = (88 \text{ ft/s})(6 \text{ s}) = 528 \text{ ft}$

h. The car is traveling at a constant speed and is moving in the positive direction. It moves the same distance in each one-second time interval.



2. A car is initially at rest. Its velocity changes by 10 mi/hr each second, ie its acceleration is (10mi/hr)/s. What is the car's velocity after:

- a. 1 s
- b. 2 s
- c. 3 s
- d. 4 s
- e. 5 s
- f. 6 s
- g. Draw a graphical representation of these results.
- h. Describe the motion of the car.

Solution

2.

a. $v_f = v_i + (a) (\Delta t)$

$$v = 0 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(1 \text{ s}) = 10 \text{ mi/hr}$$

b. $v_f = v_i + (a) (\Delta t)$

$$v = 0 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(2 \text{ s}) = 20 \text{ mi/hr}$$

c. $v_f = v_i + (a) (\Delta t)$

$$v = 0 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(3 \text{ s}) = 30 \text{ mi/hr}$$

d. $v_f = v_i + (a) (\Delta t)$

$$v = 0 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(4 \text{ s}) = 40 \text{ mi/hr}$$

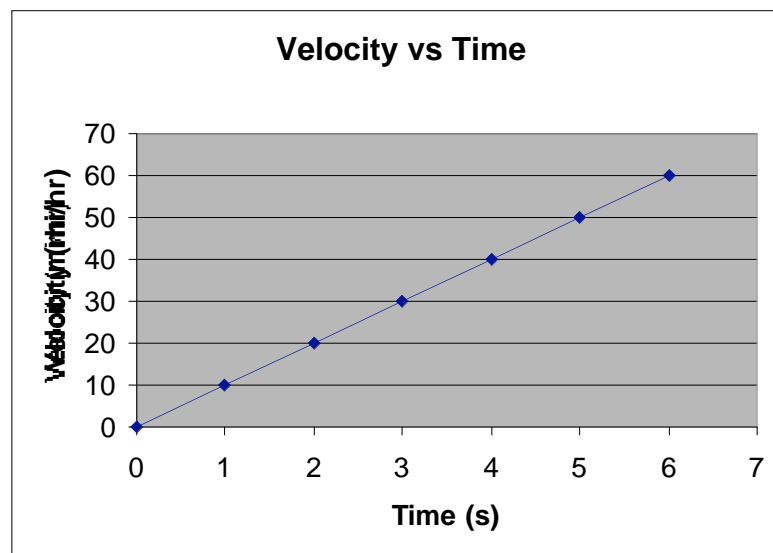
e. $v_f = v_i + (a) (\Delta t)$

$$v = 0 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(5 \text{ s}) = 50 \text{ mi/hr}$$

f. $v_f = v_i + (a) (\Delta t)$

$$v = 0 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(6 \text{ s}) = 60 \text{ mi/hr}$$

h. The car is moving faster as time passes. Its speed is increasing by 10 mi/hour every second. It is always traveling in the positive direction.



3. A car is initially at a position of $x = 0$. It is traveling in the negative direction at a speed of 88 ft/s - in other words it is traveling at a velocity of -88ft/s.. What is the car's position after:

- a. 1 s
- b. 2 s
- c. 3 s
- d. 4 s
- e. 5 s
- f. 6 s
- g. Draw a graphical representation of these results.
- i. Describe the motion of the car.

Solution:

3.

a. $x_f = x_i + vt$

$x = 0 \text{ ft} + vt = (-88 \text{ ft/s})(1 \text{ s}) = -88 \text{ ft}$

b. $x = 0 \text{ ft} + vt = (-88 \text{ ft/s})(2 \text{ s}) = -176 \text{ ft}$

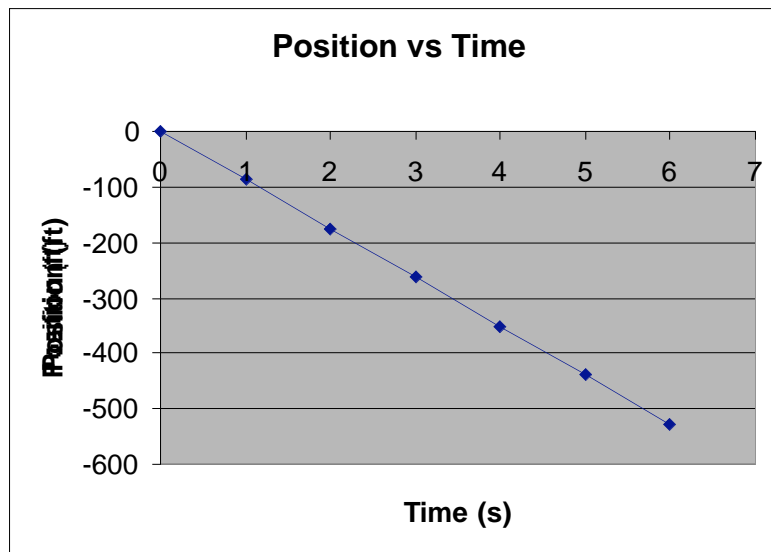
c. $x = 0 \text{ ft} + vt = (-88 \text{ ft/s})(3 \text{ s}) = -264 \text{ ft}$

d. $x = 0 \text{ ft} + vt = (-88 \text{ ft/s})(4 \text{ s}) = -352 \text{ ft}$

e. $x = 0 \text{ ft} + vt = (-88 \text{ ft/s})(5 \text{ s}) = -440 \text{ ft}$

f. $x = 0 \text{ ft} + vt = (-88 \text{ ft/s})(6 \text{ s}) = -528 \text{ ft}$

i. The car is traveling at a constant speed and is moving in the negative direction. It moves the same distance in each one-second time interval.



4. A car is initially at rest. Its speed changes by -10 mi/hr each second, ie its acceleration is (-10mi/hr)/s. What is the car's velocity after:
- 1 s
 - 2 s
 - 3 s
 - 4 s
 - 5 s
 - 6 s
 - Draw a graphical representation of these results.
 - Describe the motion of the car.

Solution

4.

a. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(1 \text{ s}) = -10 \text{ mi/hr}$$

b. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(2 \text{ s}) = -20 \text{ mi/hr}$$

c. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(3 \text{ s}) = -30 \text{ mi/hr}$$

d. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(4 \text{ s}) = -40 \text{ mi/hr}$$

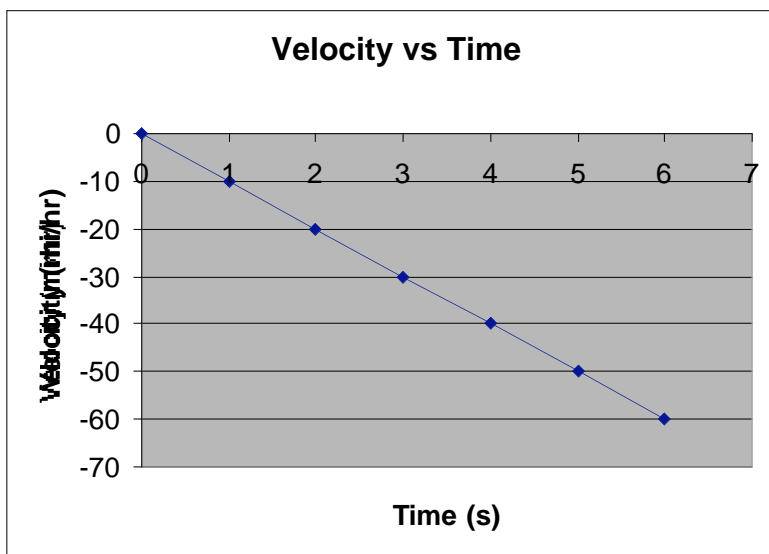
e. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(5 \text{ s}) = -50 \text{ mi/hr}$$

f. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(6 \text{ s}) = -60 \text{ mi/hr}$$

h. The car is moving faster as time passes. Its speed is increasing by 10 mi/hour every second. It is always traveling in the negative direction.



5. A car is initially at a position of $x=528$ ft.. It is traveling with a velocity of -88 ft/s. What is the car's position after:

- a. 1 s
- b. 2 s
- c. 3 s
- d. 4 s
- e. 5 s
- f. 6 s
- g. 7 s
- h. 8 s
- i. 9 s
- j. 10 s
- k. 11 s
- l. 12 s
- m. Draw a graphical representation of these results.
- n. Describe the motion of the car.

Solution:

5.

a. $x_f = x_i + vt$

$x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(1 \text{ s}) = 440 \text{ ft}$

b. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(2 \text{ s}) = 352 \text{ ft}$

c. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(3 \text{ s}) = 264 \text{ ft}$

d. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(4 \text{ s}) = 176 \text{ ft}$

e. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(5 \text{ s}) = 88 \text{ ft}$

f. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(6 \text{ s}) = 0 \text{ ft}$

g. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(7 \text{ s}) = -88 \text{ ft}$

h. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(8 \text{ s}) = -176 \text{ ft}$

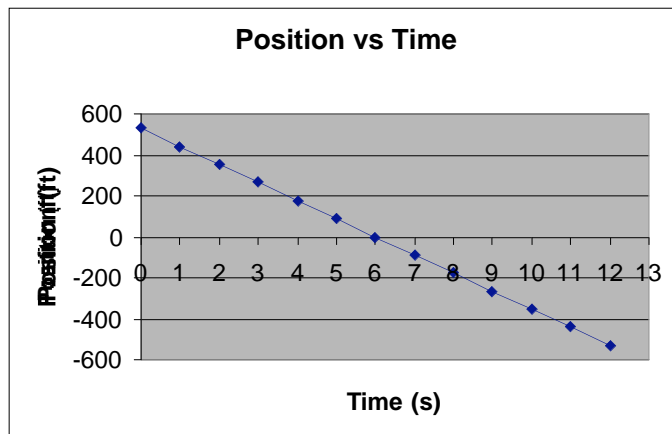
i. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(9 \text{ s}) = -264 \text{ ft}$

j. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(10 \text{ s}) = -352 \text{ ft}$

k. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(11 \text{ s}) = -440 \text{ ft}$

l. $x = 528 \text{ ft} + vt = 528 \text{ ft} + (-88 \text{ ft/s})(12 \text{ s}) = -528 \text{ ft}$

n. The car starts out at the 528 ft mark and then passes the origin and moves into negative territory. It travels at a constant speed and it is always moving in the negative direction. It moves the same distance in each one-second time-interval.



6. A car is initially traveling at 60 mi/hr. Its velocity changes by -10 mi/hr each second, ie its acceleration is (-10mi/hr)/s. What is the car's velocity after:

- a. 1 s
- b. 2 s
- c. 3 s
- d. 4 s
- e. 5 s
- f. 6 s
- g. 7 s
- h. 8 s
- i. 9 s
- j. 10 s
- k. 11 s
- l. 12 s
- m. Draw a graphical representation of these results.
- n. Describe the motion of the car.

Solution

6.

a. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(1 \text{ s}) = 50 \text{ mi/hr}$$

b. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(2 \text{ s}) = 40 \text{ mi/hr}$$

c. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(3 \text{ s}) = 30 \text{ mi/hr}$$

d. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(4 \text{ s}) = 20 \text{ mi/hr}$$

e. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(5 \text{ s}) = 10 \text{ mi/hr}$$

f. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(6 \text{ s}) = 0 \text{ mi/hr}$$

g. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(7 \text{ s}) = -10 \text{ mi/hr}$$

h. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(8 \text{ s}) = -20 \text{ mi/hr}$$

i. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(9 \text{ s}) = -30 \text{ mi/hr}$$

j. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(10 \text{ s}) = -40 \text{ mi/hr}$$

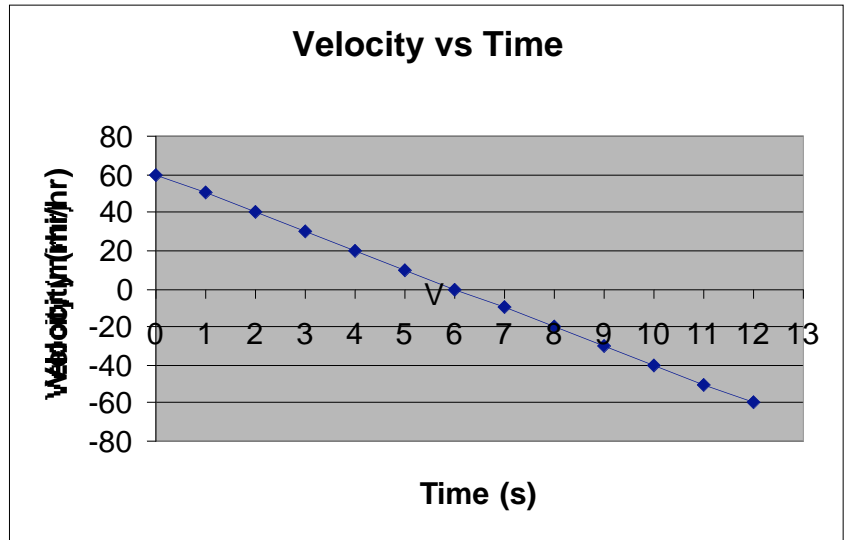
k. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(11 \text{ s}) = -50 \text{ mi/hr}$$

l. $v_f = v_i + (a) (\text{delta } t)$

$$v = 60 \text{ mi/hr} + ((-10\text{mi/hr})/\text{s})(12 \text{ s}) = -60 \text{ mi/hr}$$

n. The car is slowing down until it stops. Then it starts to speed up, traveling in the opposite direction. It ends up traveling in the negative direction.



7. A car is initially at a position of $x = -528$ ft. It is at a velocity of 88 ft/s. What is the car's position after:

- a. 1 s
- b. 2 s
- c. 3 s
- d. 4 s
- e. 5 s
- f. 6 s
- g. 7 s
- h. 8 s
- i. 9 s
- j. 10 s
- k. 11 s
- l. 12 s
- m. Draw a graphical representation of these results.
- n. Describe the motion of the car.

Solution:

7.

a. $x_f = x_i + vt$

$x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(1 \text{ s}) = -440 \text{ ft}$

b. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(2 \text{ s}) = -352 \text{ ft}$

c. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(3 \text{ s}) = -264 \text{ ft}$

d. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(4 \text{ s}) = -176 \text{ ft}$

e. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(5 \text{ s}) = -88 \text{ ft}$

f. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(6 \text{ s}) = 0 \text{ ft}$

g. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(7 \text{ s}) = 88 \text{ ft}$

h. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(8 \text{ s}) = 176 \text{ ft}$

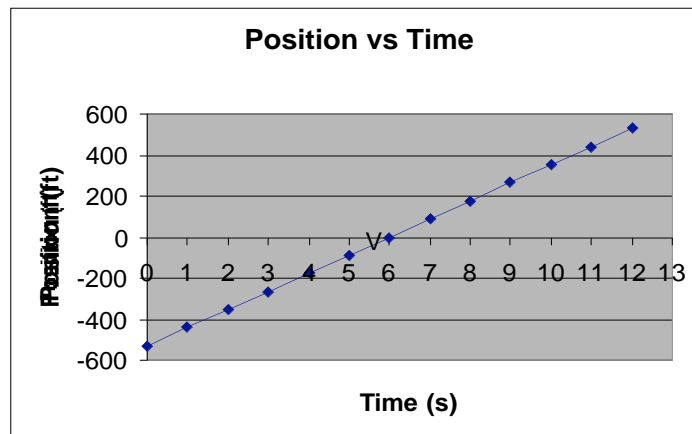
i. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(9 \text{ s}) = 264 \text{ ft}$

j. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(10 \text{ s}) = 352 \text{ ft}$

k. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(11 \text{ s}) = 440 \text{ ft}$

l. $x = -528 \text{ ft} + vt = -528 \text{ ft} + (88 \text{ ft/s})(12 \text{ s}) = 528 \text{ ft}$

n. The car starts out at the -528 ft mark and then passes the origin and moves into positive territory. It travels at a constant speed and it is always moving in the positive direction. It moves the same distance in each one-second time-interval.



8. A car is initially traveling at -60 mi/hr. Its speed changes by 10 mi/hr each second, ie its acceleration is (10mi/hr)/s. What is the car's speed after:

- a. 1 s
- b. 2 s
- c. 3 s
- d. 4 s
- e. 5 s
- f. 6 s
- g. 7 s
- h. 8 s
- i. 9 s
- j. 10 s
- k. 11 s
- l. 12 s
- m. Draw a graphical representation of these results.
- n. Describe the motion of the car.

Solution

8.

a. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(1 \text{ s}) = -50 \text{ mi/hr}$$

b. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(2 \text{ s}) = -40 \text{ mi/hr}$$

c. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(3 \text{ s}) = -30 \text{ mi/hr}$$

d. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(4 \text{ s}) = -20 \text{ mi/hr}$$

e. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(5 \text{ s}) = -10 \text{ mi/hr}$$

f. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(6 \text{ s}) = 0 \text{ mi/hr}$$

g. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(7 \text{ s}) = 10 \text{ mi/hr}$$

h. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(8 \text{ s}) = 20 \text{ mi/hr}$$

i. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(9 \text{ s}) = 30 \text{ mi/hr}$$

j. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(10 \text{ s}) = 40 \text{ mi/hr}$$

k. $v_f = v_i + (a) (\text{delta } t)$

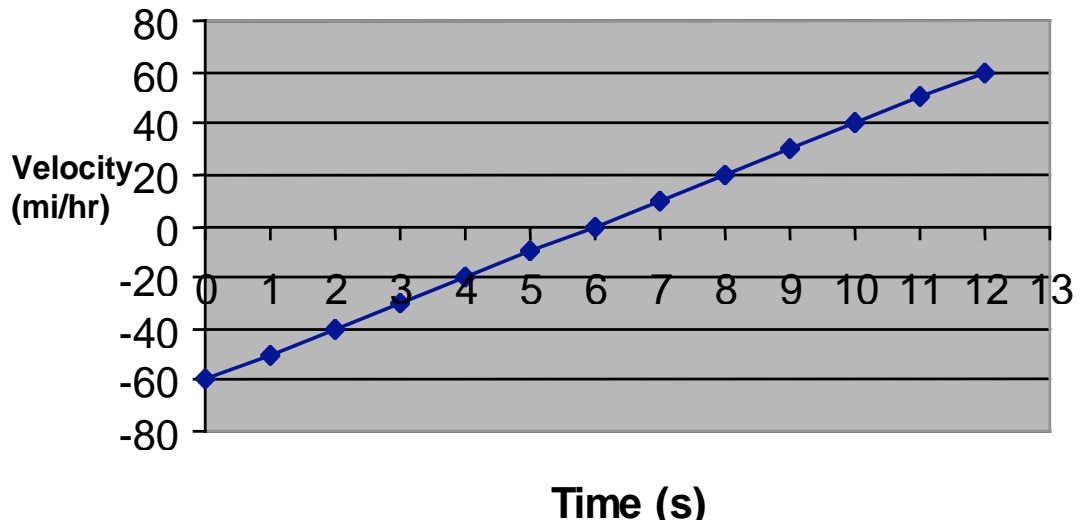
$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(11 \text{ s}) = 50 \text{ mi/hr}$$

l. $v_f = v_i + (a) (\text{delta } t)$

$$v = -60 \text{ mi/hr} + ((10\text{mi/hr})/\text{s})(12 \text{ s}) = 60 \text{ mi/hr}$$

n. The car is slowing down until it stops. Then it starts to speed up, traveling in the opposite direction. It ends up traveling in the positive direction.

Velocity vs Time



9. A skydiver steps off a tall cliff. Her speed changes by -10m/s every second. i.e. her acceleration is $(-10\text{m/s})/\text{s}$. What is her speed after:

- a. 0 s
- b. 1 s
- c. 2 s
- d. 3 s
- e. 4 s
- f. 5 s
- g. Draw a graphical representation of these results.
- h. Describe the motion of the skydiver.

Solution

9.

a. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ m/s} + ((-10\text{m/s})/\text{s})(0 \text{ s}) = 0 \text{ m/s}$$

b. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ m/s} + ((-10\text{m/s})/\text{s})(1 \text{ s}) = -10 \text{ m/s}$$

c. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ m/s} + ((-10\text{m/s})/\text{s})(2 \text{ s}) = -20 \text{ m/s}$$

d. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ m/s} + ((-10\text{m/s})/\text{s})(3 \text{ s}) = -30 \text{ m/s}$$

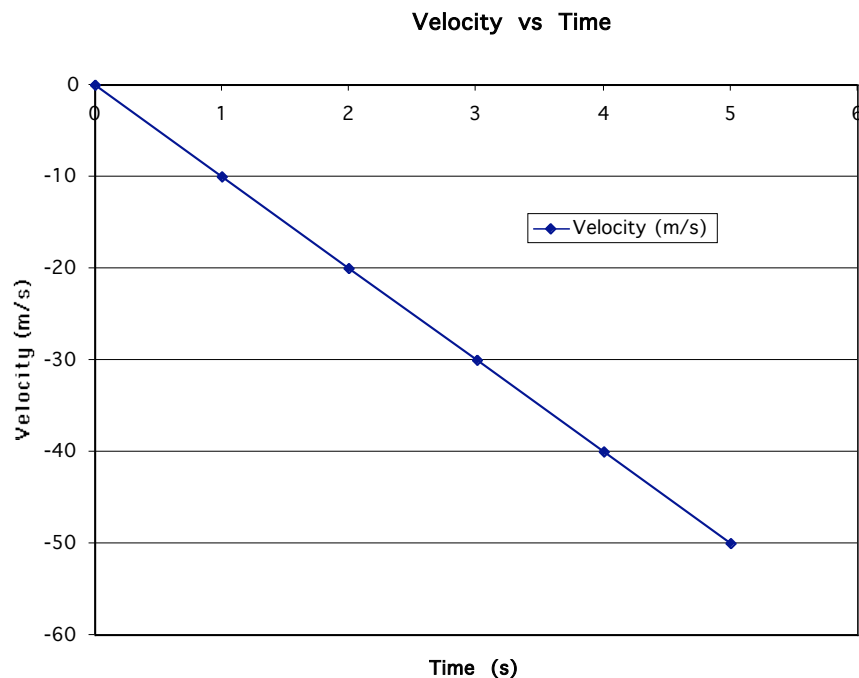
e. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ m/s} + ((-10\text{m/s})/\text{s})(4 \text{ s}) = -40 \text{ m/s}$$

f. $v_f = v_i + (a) (\text{delta } t)$

$$v = 0 \text{ m/s} + ((-10\text{m/s})/\text{s})(5 \text{ s}) = -50 \text{ m/s}$$

h. The skydiver starts from rest and then picks up speed as time goes on, moving faster and faster. The skydiver is always falling towards the earth.



For the skydiver in problem 9, calculate how far she travels each second and the direction of travel - use the average speed for each one second interval.

aa. Between 0 and 1 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (0\text{m/s} + -10\text{m/s})/2 \times 1 \text{ s} = -5\text{m}$ (downward)

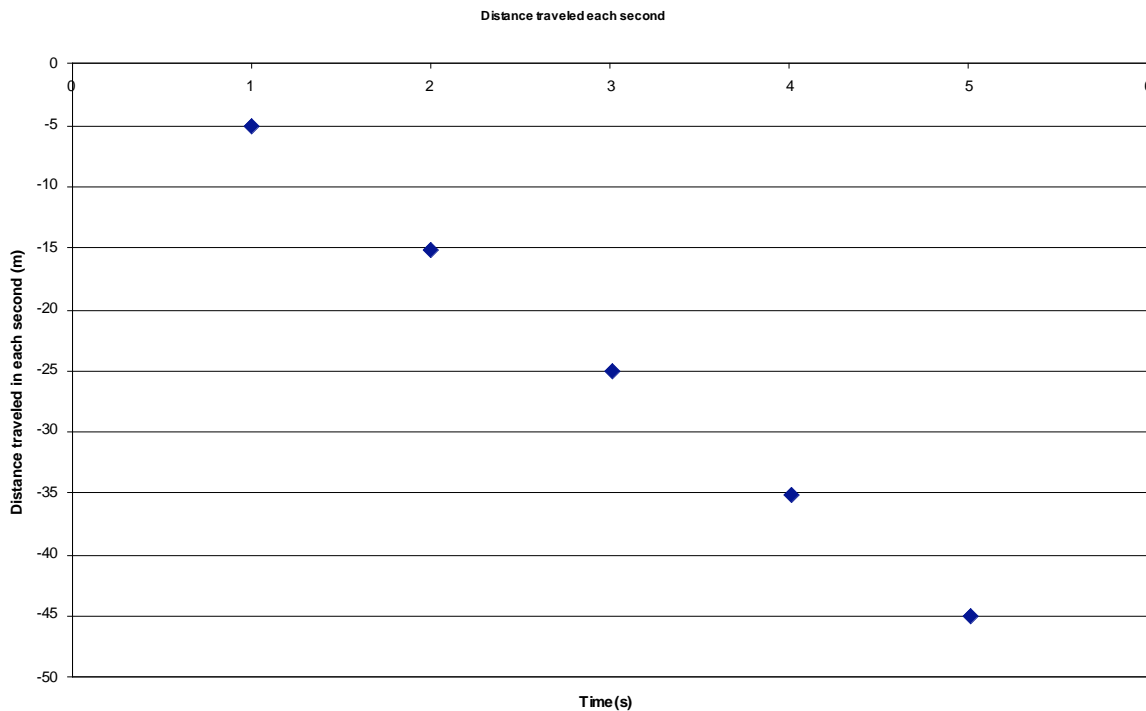
bb. Between 1 and 2 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-10\text{m/s} + -20\text{m/s})/2 \times 1 \text{ s} = -15\text{m}$ (downward)

cc. Between 2 and 3 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-20\text{m/s} + -30\text{m/s})/2 \times 1 \text{ s} = -25\text{m}$ (downward)

dd. Between 3 and 4 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-30\text{m/s} + -40\text{m/s})/2 \times 1 \text{ s} = -35\text{m}$ (downward)

ee. Between 4 and 5 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-40\text{m/s} + -50\text{m/s})/2 \times 1 \text{ s} = -45\text{m}$ (downward)

ff. Plot the average velocity of the skydiver at 1-second intervals.



For the skydiver in problem 9, calculate the distance below the cliff edge that the skydiver has fallen after each 1 s interval.

Call the position of the cliff at $x=0$

aaa. After 1 s: Height = x initial + $\Delta x = 0\text{m} - 5\text{m} = -5\text{ m}$

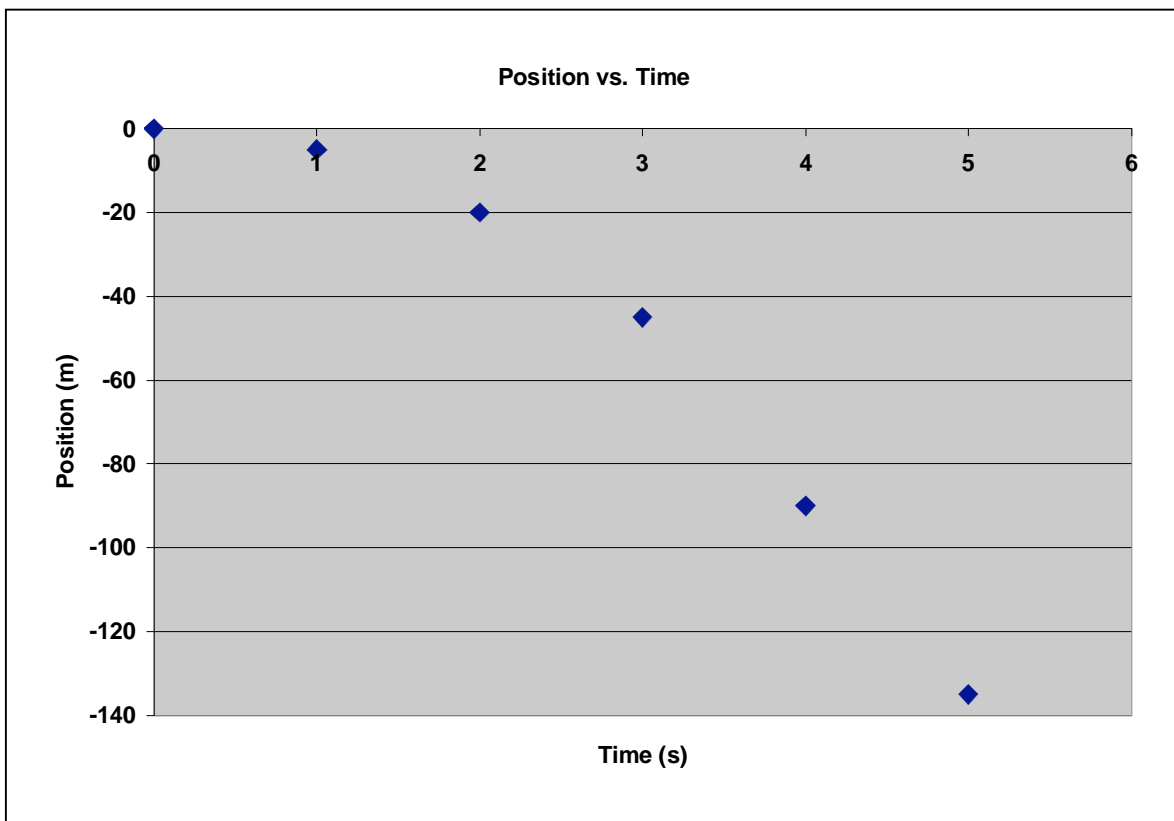
bbb. After 2 s: Height = x initial + $\Delta x = -5\text{m} - 15\text{m} = -20\text{ m}$

ccc. After 3 s: Height = x initial + $\Delta x = -20\text{m} - 25\text{m} = -45\text{ m}$

ddd. After 4 s: Height = x initial + $\Delta x = -45\text{m} - 35\text{m} = -90\text{ m}$

eee. After 5 s: Height = x initial + $\Delta x = -90\text{m} - 45\text{m} = -135\text{ m}$

kkk. Plot the position of the skydiver below the cliff edge in 1 second intervals.



10. A toy rocket is launched from the ground with an initial velocity upward of 50m/s. Its velocity changes by (-10m/s) every second, i.e. its acceleration is (-10m/s)/s. What is its speed after:

- a. 0 s
- b. 1 s
- c. 2 s
- d. 3 s
- e. 4 s
- f. 5 s
- g. 6 s
- h. 7 s
- i. 8 s
- j. 9 s
- k. 10 s

- l. Draw a graphical representation of these results.
- m. Describe the motion of the toy rocket.

Solution:

10.

a. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(0 \text{ s}) = 50 \text{ m/s}$$

b. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(1 \text{ s}) = 40 \text{ m/s}$$

c. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(2 \text{ s}) = 30 \text{ m/s}$$

d. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(3 \text{ s}) = 20 \text{ m/s}$$

e. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(4 \text{ s}) = 10 \text{ m/s}$$

f. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(5 \text{ s}) = 0 \text{ m/s}$$

g. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(6 \text{ s}) = -10 \text{ m/s}$$

h. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(7 \text{ s}) = -20 \text{ m/s}$$

i. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(8 \text{ s}) = -30 \text{ m/s}$$

j. $v_f = v_i + (a) (\text{delta } t)$

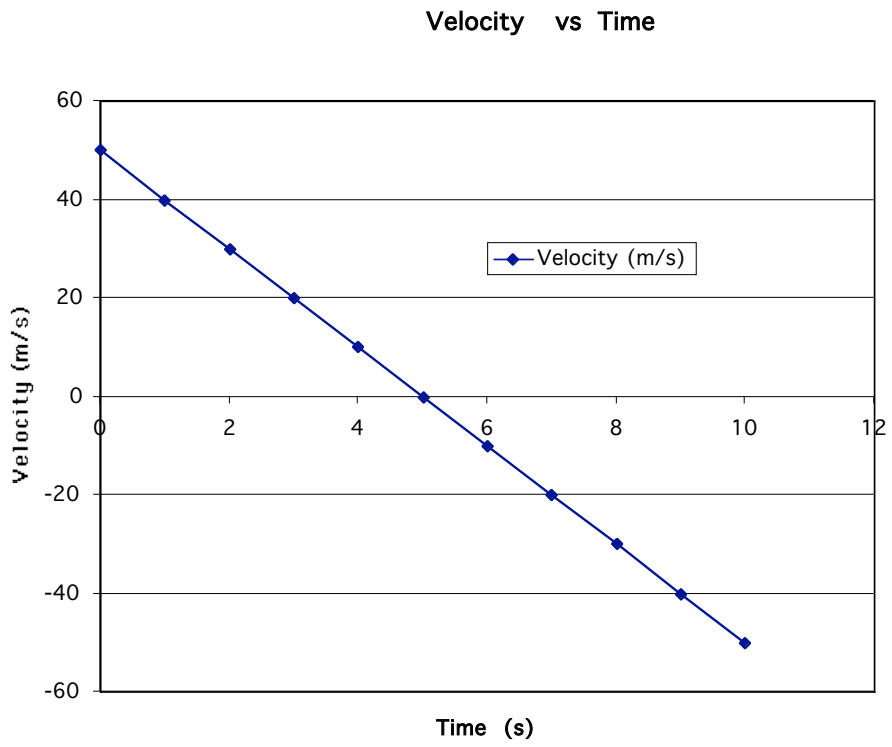
$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(9 \text{ s}) = -40 \text{ m/s}$$

k. $v_f = v_i + (a) (\text{delta } t)$

$$v = 50 \text{ m/s} + ((-10\text{m/s})/\text{s})(10 \text{ s}) = -50 \text{ m/s}$$

m. The toy rocket is moving upwards, but is slowing down until it stops. Then it starts to speed up, traveling downward towards the earth.

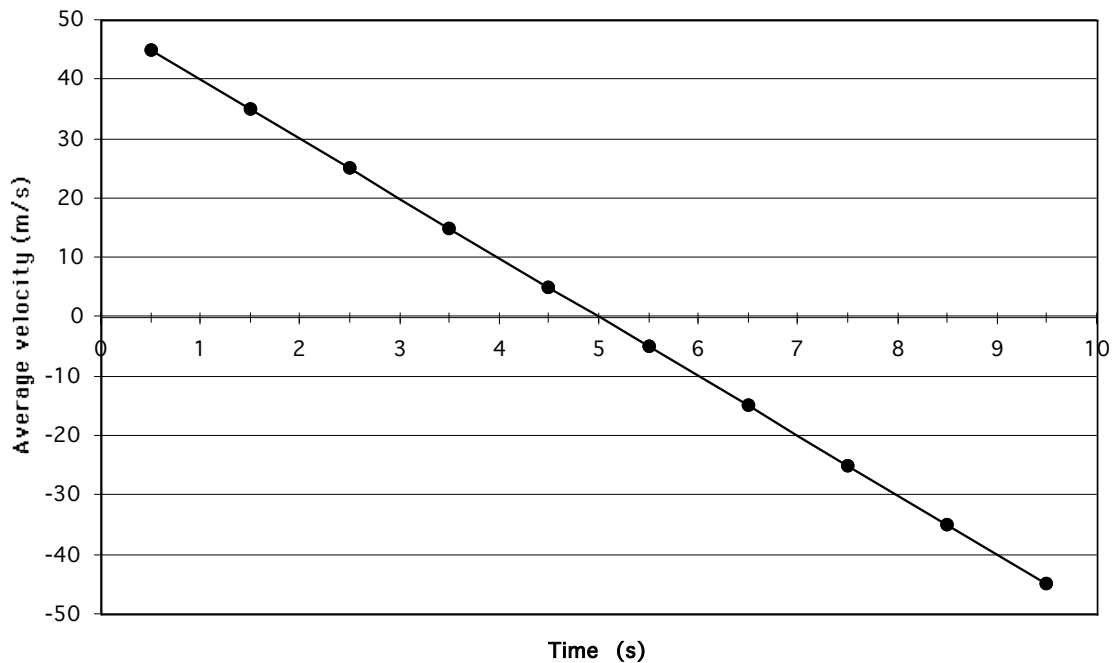
1.



For the toy rocket in problem 10, calculate how far it travels each second and the direction of travel - use the average speed for each one second interval.

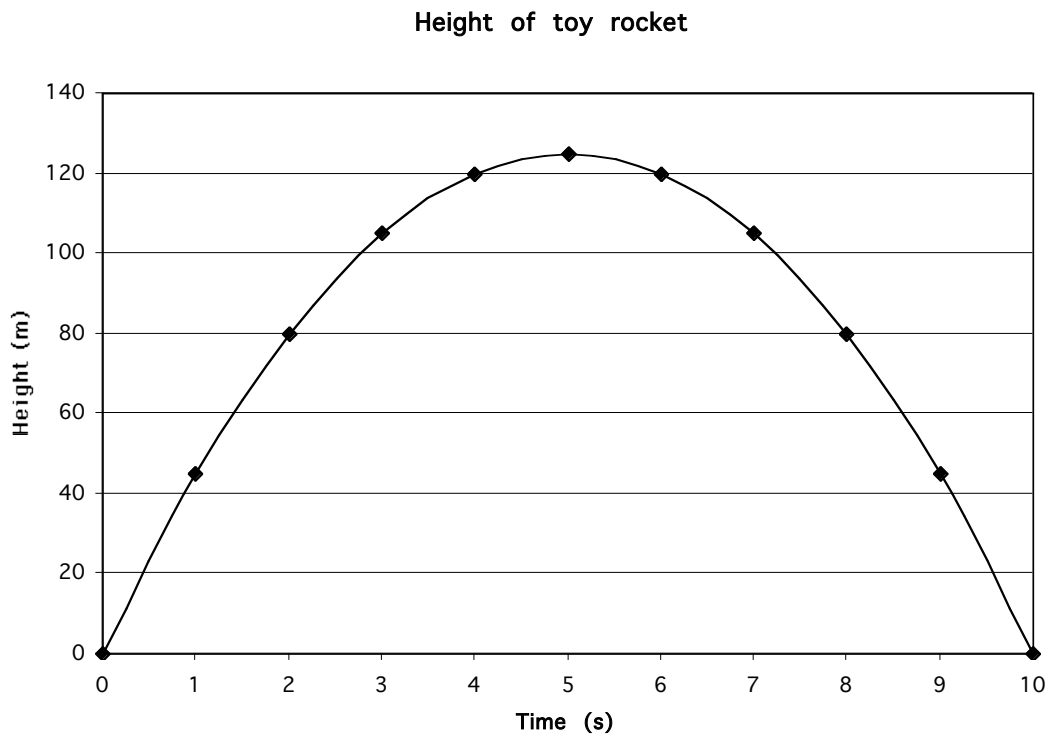
- aa. Between 0 and 1 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (50\text{m/s} + 40\text{m/s})/2 \times 1 \text{ s} = 45\text{m}$ upward
- bb. Between 1 and 2 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (40\text{m/s} + 30\text{m/s})/2 \times 1 \text{ s} = 35\text{m}$ (upward)
- cc. Between 2 and 3 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (30\text{m/s} + 20\text{m/s})/2 \times 1 \text{ s} = 25\text{m}$ (upward)
- dd. Between 3 and 4 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (20\text{m/s} + 10\text{m/s})/2 \times 1 \text{ s} = 15\text{m}$ (upward)
- ee. Between 4 and 5 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (10\text{m/s} + 0\text{m/s})/2 \times 1 \text{ s} = 5\text{m}$ (upward)
- ff. Between 5 and 6 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (0\text{m/s} - 10\text{m/s})/2 \times 1 \text{ s} = -5\text{m}$ (downward)
- gg. Between 6 and 7 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-10\text{m/s} - 20\text{m/s})/2 \times 1 \text{ s} = -15\text{m}$ (downward)
- hh. Between 7 and 8 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-20\text{m/s} - 30\text{m/s})/2 \times 1 \text{ s} = -25\text{m}$ (downward)
- ii. Between 8 and 9 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-30\text{m/s} - 40\text{m/s})/2 \times 1 \text{ s} = -35\text{m}$ (downward)
- jj. Between 9 and 10 s: $\Delta x = v_{\text{avg}} \times 1 \text{ s} = (-40\text{m/s} - 50\text{m/s})/2 \times 1 \text{ s} = -45\text{m}$ (downward)
- kk. Plot the velocity of the rocket at 1 second intervals.

Average Velocity vs Time for Toy Rocket



For the toy rocket in problem 10, calculate the height of the rocket after each 1 s interval.

- aaa. After 1 s: Height = $x_{\text{initial}} + \Delta x = 0\text{m} + 45\text{m} = 45\text{ m}$
- bbb. After 2 s: Height = $x_{\text{initial}} + \Delta x = 45\text{m} + 35\text{m} = 80\text{ m}$
- ccc . After 3 s: Height = $x_{\text{initial}} + \Delta x = 80\text{m} + 25\text{m} = 105\text{ m}$
- ddd. After 4 s: Height = $x_{\text{initial}} + \Delta x = 105\text{m} + 15\text{m} = 120\text{ m}$
- eee. After 5 s: Height = $x_{\text{initial}} + \Delta x = 120\text{m} + 5\text{m} = 125\text{ m}$
- fff. After 6 s: Height = $x_{\text{initial}} + \Delta x = 125\text{m} - 5\text{m} = 120\text{ m}$
- ggg. After 7 s: Height = $x_{\text{initial}} + \Delta x = 120\text{m} - 15\text{m} = 105\text{ m}$
- hhh. After 8 s: Height = $x_{\text{initial}} + \Delta x = 105\text{m} - 25\text{m} = 80\text{ m}$
- iii. After 9 s: Height = $x_{\text{initial}} + \Delta x = 80\text{m} - 35\text{m} = 45\text{ m}$
- jjj. After 10 s: Height = $x_{\text{initial}} + \Delta x = 45\text{m} - 45\text{m} = 0\text{ m}$
- kkk. Plot the height of the rocket in 1 second intervals.



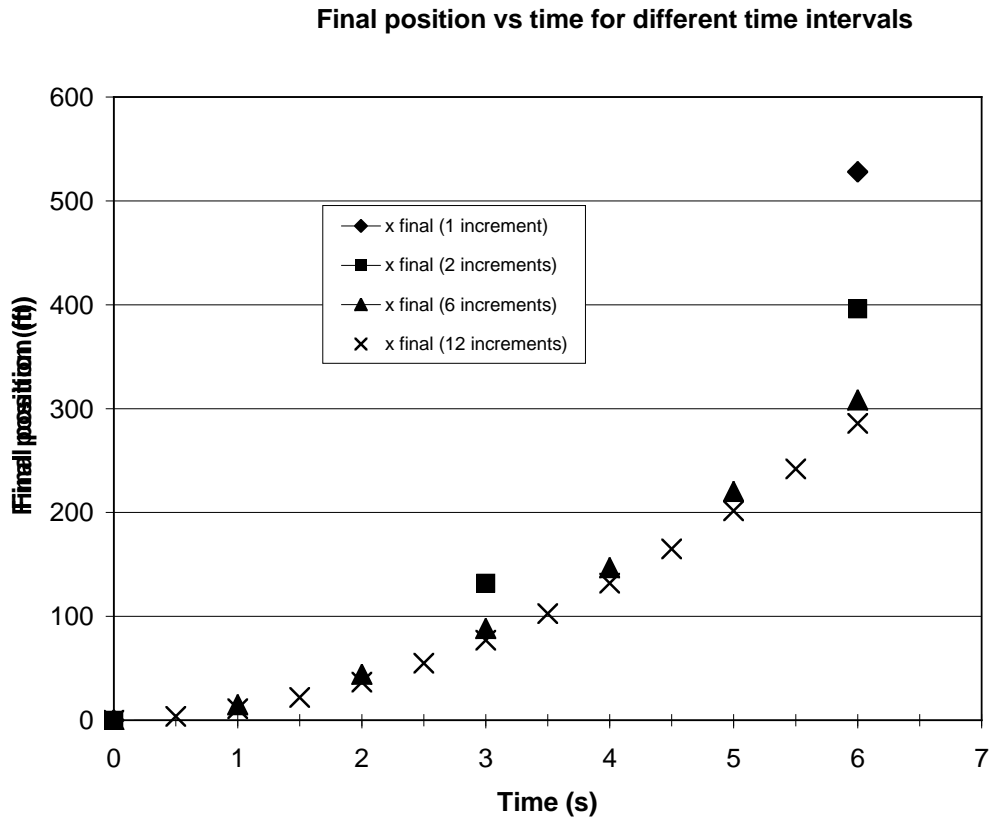
11. Let's investigate the relationship between position, speed, and acceleration in a very straightforward way. Recall that for an object moving with constant acceleration, the speed changes by the same amount each time period that you choose. And for constant speed, the distance traveled by an object is the same in each time period that you choose.

So figure out the acceleration, speed change, final speed, distance traveled and final distance for a situation with decreasing time intervals. Assume that a car is accelerating at from 0 to 60 mi/hr in 6 sec. Recall that 60 mi/hr is 88 ft/s. Make a spreadsheet chart of these parameters using the following number of time intervals:

- 1
- 2
- 6
- 12
- Make a graph of the position of the car vs time for each number of time intervals.
- What do you notice about the graph? Is it linear or quadratic? Does it seem to converge to a certain curve as the time intervals get shorter and shorter?
- Can you demonstrate that there is a mathematical relationship between distance traveled at constant acceleration for a given interval of time and the data you generated in this table? Answers a-d in table below.

t (s)	delta t (s)	delta v (ft/s)	v final (ft/s)	delta x (ft)	x final (ft)
	(=6 s/increments)	(=(88ft/s)/increments)	(= v previous + delta v)	(=v final * delta t)	(= x previous + delta x)
	1 increment				
0					
6	6	88	88	528	528
	2 increments				
0					
3	3	44	44	132	132
6	3	44	88	264	396
	6 increments				
0					
1	1	14.67	14.67	14.67	14.674
2	1	14.67	29.34	29.34	44.014
3	1	14.67	44.01	44.01	88.024
4	1	14.67	58.68	58.68	146.704
5	1	14.67	73.35	73.35	220.054
6	1	14.67	88.02	88.02	308.074
	12 increments				
0					
0.5	0.5	7.33	7.33	3.665	3.665
1	0.5	7.33	14.66	7.33	10.995
1.5	0.5	7.33	21.99	10.995	21.99
2	0.5	7.33	29.32	14.66	36.65
2.5	0.5	7.33	36.65	18.325	54.975
3	0.5	7.33	43.98	21.99	76.965
3.5	0.5	7.33	51.31	25.655	102.62
4	0.5	7.33	58.64	29.32	131.94
4.5	0.5	7.33	65.97	32.985	164.925
5	0.5	7.33	73.3	36.65	201.575
5.5	0.5	7.33	80.63	40.315	241.89
6	0.5	7.33	87.96	43.98	285.87

e.



f. The graph appears to be quadratic, not linear.

$$\begin{aligned} g. x &= 0.5 a t^2 = 0.5 (10\text{mi}/(\text{hr}\cdot\text{s})) (6\text{s})^2 \\ &= 180\text{mi}\cdot\text{s}/\text{hr} \times (1\text{hr}/3600\text{s}) \times (5280\text{ft}/\text{mi}) \\ &= 264 \text{ ft} \end{aligned}$$

This is very close to the value of 286 ft that we obtained using 12 increments of time in part d.

12. Show that $x = v_i t + 0.5at^2$ is equivalent to $v_f^2 - v_i^2 = 2ax$. Recall that $v_f = v_i + at$, so $t = (v_f - v_i)/a$

$$\begin{aligned} x &= v_i t + 0.5at^2 \\ &= v_i (v_f - v_i)/a + 0.5a(v_f - v_i)^2/a^2 \\ &= (v_f - v_i) \{v_i/a + 0.5(v_f - v_i)/a\} \\ &= (v_f - v_i) \{0.5(v_f + v_i)/a\} \\ &= 0.5 (v_f^2 - v_i^2)/a \end{aligned}$$

So $v_f^2 - v_i^2 = 2ax$

13. Begin with the equation $v_f^2 - v_i^2 = 2ax$.

a. Multiply both sides of the equation by $0.5m$, where m is the mass of an object moving with speed v .

$$0.5mv_f^2 - 0.5mv_i^2 = max$$

b. Now define the initial kinetic energy of the object as $0.5mv_i^2$ and define the final kinetic energy of the object as $0.5mv_f^2$ and define the force on the object as ma and recall that x is the distance traveled by the object. Rewrite the equation in part a using these definitions.

$$\text{final kinetic energy} - \text{initial kinetic energy} = \text{force} * \text{distance}$$

Since $\text{force} * \text{distance} = \text{work}$, this shows that work is equal to the change in kinetic energy.

14. Again consider the equation $0.5mv_f^2 - 0.5mv_i^2 = max$ from problem 13.

a. Now consider an object free falling near the earth and call this constant acceleration g . Let the distance the object falls be called h . Rewrite the above equation.

$$\text{final kinetic energy} - \text{initial kinetic energy} = mgh$$

b. Now define a term called potential energy difference as mgh . Rewrite the above equation.

$$\text{final kinetic energy} - \text{initial kinetic energy} = \text{potential energy difference}$$

or $(\text{final kinetic energy} - \text{initial kinetic energy}) - \text{potential energy difference} = 0$

or $(\text{change in kinetic energy}) - (\text{change in potential energy}) = 0$: Energy Conservation!

Note that the law of energy conservation is a direct consequence of the time translational symmetry of nature. In other words, since nature's laws work the same today as they did yesterday or will tomorrow, then as a result we get the law of the conservation of energy. For further details about the relationship between symmetry and conservation laws, see the wonderful web site: <http://www.emmynoether.com/noeth.htm>

15. Consider the possible paths A, B, C, D an object may take to get from one point to another point in the same amount of time - in this case 10 m from the starting point in 10 seconds. It turns out that it will follow the path that has the minimum value of the sum of the (product of the kinetic energy ($0.5mv^2$) and the time interval) over the path. This value is called the "action." (We are neglecting gravity. If gravity is present, we must include the potential energy in some way.) Assume a mass of 1kg. The "Principle of Least Action" is another way of formulating the laws of motion in the same way that the Principle of Least Time (also known as Fermat's Principle) is another way of formulating the laws governing the behavior of light.

a. Calculate the "action" for each of the 4 paths. Which path will the object follow?

Path A: Action = $0.5 (1\text{kg}) (1\text{m/s})^2 * 10 \text{ s} = 5 \text{ kg m}^2/\text{s}$

Path B: Action = $0.5 (1\text{kg}) (10\text{m/s})^2 * 1\text{ s} = 50 \text{ kg m}^2/\text{s}$

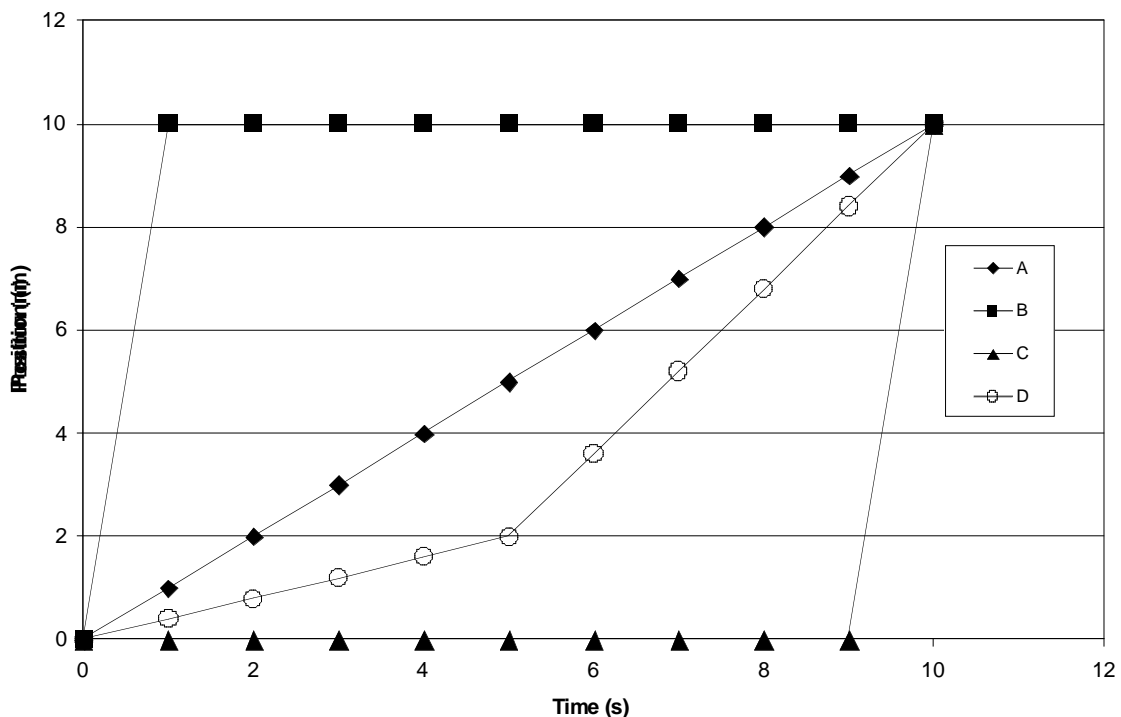
Path C: Action = $0.5 (1\text{kg}) (10\text{m/s})^2 * 1\text{ s} = 50 \text{ kg m}^2/\text{s}$

Path D: Action = $0.5 (1\text{kg}) (2\text{m}/5\text{s})^2 * 5 \text{ s} + 0.5 (1\text{kg}) (8\text{m}/5\text{s})^2 * 5 \text{ s} = 6.8 \text{ kg m}^2/\text{s}$

The object will follow path A, because that is the path of Least Action. It is the path of an object moving at constant speed.

(For a beautiful discussion of this topic - for advanced students - see the Feynman Lectures on Physics, Volume 1, Chapter 19: "The Principle of Least Action.")

Possible particle paths



b. Describe the motions of the objects traveling along these "paths."

A: This object travels at a constant speed of 1m/s from 0 to 10 s.

B. This object travels 10 m in 1 s, and then stops.

C. This object does not move for 9 s, and then travels 10m in 1s.

D. This object travels 2 m in 5 s, then 8 m in 5 s, so it moves faster during the time interval of 5-10 s.