The Line of Resistance

Using a Graphite Pencil to Explore the Electrical Properties of Materials and Circuits

Institute for Chemical Education

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ICE Publication 97-002

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Preface

Since 1992, General Atomics (GA) has received wide recognition for its Science Education Outreach Program, a volunteer effort of GA employees and San Diego science teachers. The goals of the program are to improve the quality of science education and to encourage more students to pursue science careers. In order to expand the program, the GA Sciences Education Foundation was established in 1995.

The General Atomics Sciences Education Foundation is committed to playing a major role to enhance pre-college education in science, engineering and new technologies. This commitment is being realized through workshops using materials developed by scientists at GA. Hands-on learning, teamwork using scientific processes, and abstract and conceptual thinking are fundamental to the design of the workshops. This approach to instruction meets the guidelines of both the *National Science Education Standards* and the *Benchmarks for Science Literacy.*

The Line of Resistance was developed by Dr. Lawrence Woolf of GA. His idea for the activities originated with an attempt to measure the resistance of pencil lead during an Exploration of Materials Science workshop. The ability to simply draw lines and circuits and measure their resistances was immediately appealing to everyone in the workshop; however, initial attempts to utilize the power and simplicity of this approach were stymied by the irreproducible results obtained by using the standard No. 2 pencil. Since the pure graphite pencil resolved this problem, *The Line of Resistance* has been tested in workshops and refined using input from participating teachers, GA staff, faculty at the University of Wisconsin, and staff of the Institute for Chemical Education. It is intended for use as supplemental curriculum primarily in grades 8–12, but can be adapted for use in middle school and college. Familiarity with basic electrical principles and terminology is important to use this activity most effectively with students.

A partnership between the Institute for Chemical Education (ICE) and the General Atomics Sciences Education Foundation makes publication of *The Line of Resistance* possible. As always, ICE encourages feedback on its publications—we welcome corrections, additions, tips for improving classroom use, suggestions for curriculum integration, etc. Please forward your input to us at the address on the back cover, or email it to ICE@chem.wisc.edu.

To learn more about the General Atomics Sciences Education Foundation, contact Patricia Winter, Education Outreach Coordinator, General Atomics Sciences Education Foundation, PO Box 85608, San Diego, CA 92186-9784; Phone (619) 455-3335; FAX (619) 455-3379; Pat.Winter@gat.com; http://www.sci-ed-ga.org.

With Gratitude

None of the efforts of the General Atomics (GA) Education Outreach Program would have been possible without the vision, effort, and prodding of Patricia Winter, the GA Education Outreach Coordinator. I would also like to acknowledge the constant support and encouragement of Anne Blue. Patricia and Anne initiated the program at GA and quickly formed a partnership with the San Diego-area teachers through the efforts of Nancy Taylor, Science Coordinator at the San Diego County Office of Education. They all deserve much credit for trying to improve science education in San Diego, as well as across the nation.

The support of GA management, particularly Neal Blue and Dr. Terry Gulden, is gratefully acknowledged. I would also like to acknowledge the Exploration of Materials Science teacher/GA scientist team, out of whose work this module evolved: team leader Dr. Terry Gulden, Dr. Hal Streckert, and Kirk Norton, all of GA; Joe Baron and Shauna Brammer of La Jolla High School; Roger Wynn of Mt. Empire Jr./Sr. High School; Dr. Danine Ezell of Bell Junior High School; and Paul Ziegler of Carlsbad High School. In addition, I would like to acknowledge Dr. Hal Streckert and Roger Wynn, who helped originate the idea for the module at an Exploration of Materials Science workshop at the University of Denver. I would also like to acknowledge Steve Jones and Joe Baron for initiating experiments that combined a graphite pencil and a piezoelectric transducer.

For reviewing and making numerous suggestions to improve this module, I would like to thank Dr. Hal Streckert, Dr. Terry Gulden, and my other colleagues at GA. I would also like to thank Dr. Art Ellis, Dr. David Larbalestier, and their colleagues at the National Science Foundation's Materials Research Science and Engineering Center on Nanostructured Materials and Interfaces, which is based at the University of Wisconsin–Madison.

I would like to thank Christy Cargille for the excellent job she did in transforming my rather crude document into a visually appealing and educationally appropriate module. I would also like to acknowledge the able assistance of Amy Jo Huseth and the entire ICE organization.

Finally, I would like to thank my wife Wendy for encouraging me to get more involved in community service in general and education in particular. I would also like to thank my three children: Rebecca, David, and Rachel, who assisted in early testing of many of the experiments on our kitchen table.

Dr. Lawrence D. Woolf
General Atomics, 1997

Table of Contents

About This Booklet

The purpose of *The Line of Resistance* is to provide teachers with creative, inexpensive, hands-on/minds-on ideas that illustrate electrical properties of materials and the principles of circuits. The ideas include activities, as well as creative drama exercises, intended to supplement your regular curriculum materials. The publication is directed to high school and college science teachers. Some aspects of the publication are also appropriate for middle school (after Ohm's Law has been introduced), but the mathematics and explanations may be beyond the conceptual level of most middle school students.

The booklet begins with a section on teaching strategies for *The Line of Resistance*. We offer suggestions for classroom organization, introductory exercises, and assessments. The creative drama exercises near the end of the booklet complement the teaching strategies. We encourage you to integrate the ideas and activities in these sections when you use the student activities.

A detailed teacher's guide for learning and performing the activities follows the section on teaching strategies. Once you, the teacher, have mastered the exercises and concepts, you can use them with your students. The student activities are more open-ended than the teacher's guide, allowing greater opportunity for students to construct their own knowledge of electrical properties with, of course, help from you. (We discourage providing students with the detailed teacher activities—they do not provide enough room for the students to be creative and learn for themselves, from one other, and from you.)

The student activities do not have to be performed in order, though it is best to do the "Introductory Exercise" and "The Basics of Resistance" first. Allow ample class time to explore *The Line of Resistance*. Your students will benefit more from doing a few of the activities in depth, rather than all of them in a rush.

The exploratory activities in this booklet correspond to a number of the National Science Education Standards (NSES) for physical science in middle and high school. Some of these physical science standards are listed below.

Properties and Changes of Properties in Matter

- ❖ "A substance has characteristic properties, such as density, a boiling point and solubility, all of which are independent of the amount of the sample."

Transfer of Energy

- ❖ "Energy is a property of many substances and is associated with heat, light, electricity, mechanical motion, sound, nuclei and the nature of a chemical. Energy is transferred in many ways."

- “Electrical circuits provide a means of transferring electrical energy when heat, light, sound and chemical changes are produced.”

Interaction of Energy and Matter

- “In some materials, such as metals, electrons flow easily, whereas in insulating materials such as glass they can hardly flow at all. Semiconducting materials have intermediate behavior. At low temperatures some materials become superconductors and offer no resistance to the flow of electrons.”

The activities also address the NSES standards that call for more emphasis on

- “Learning subject matter disciplines in the context of inquiry...”
- “Investigations over extended periods of time.”
- “Using multiple process skills—manipulative, cognitive, procedural.”
- “Doing more investigations in order to develop understanding, ability, values of inquiry and knowledge of science content.”

At the end of this booklet is an explanation of the scientific principles behind the activities. The explanation extends beyond the information typically found in middle and high school texts; it gives a broad summary of the properties of materials and provides detail for understanding the activities and the mathematics.

Many aspects of *The Line of Resistance* relate to students’ everyday lives, and making connections between the two can further learning. Graphite pencils are familiar to students. Drawing resistors and circuits with pencils allows the students freedom to explore and discover relationships for themselves. They may be interested to know that the conductive property of graphite has resulted in some practical real-world uses. For example, in the area of computer hardware, some printed circuit boards have been created to run a self-diagnostic procedure (i.e., a self-test) using graphite. When a graphite line is drawn to close a specific gap in the circuit board, the conduction of the graphite closes the circuit, and causes the board to run its tests. When testing is complete, the graphite mark can be erased to induce the card to resume normal behavior. Examples like these and the activities here will help students form new associations with graphite (e.g., graphite as a conductor of electricity, graphite as circuitry material).

Supplies

Listed below are descriptions of the supplies that are needed to complete all of the activities in this booklet. A sample of many of these materials has been included with the booklet so that teachers can try the activities before using them with students. Since the activities are intended for hands-on/minds-on student learning, we also tell you how to obtain additional supplies for your students.

- **Graphite pencil** (included)

  The included graphite pencil consists entirely of graphite. It is much more effective than conventional pencils for use in the activities because the insulating clay, wax, and/or polymer components are absent. Though any pure graphite pencil will work, we suggest using the PRO ART graphite (woodless) pencil. Acceptable model numbers are PRO-8911-HBC (hard), PRO-8911-4BC (soft), and PRO-8911-2BC (medium). To purchase additional graphite pencils, check art stores, office supply stores, and university bookstores. For a local PRO ART distributor in your area, call 503/643-9050.

  Note: No. 6B and other soft pencils with high graphite content will probably work, and are cheaper and more readily available alternatives to pure graphite. The standard No. 2 pencil does not provide consistent results and should be avoided.

- **Paper** (graph paper is preferred; a reproducible sheet is provided near the back of this booklet)

  Paper is needed to perform most of the activities in this publication. Graph paper is preferred because it is helpful when varying the length and width of the lines that will be drawn.

- **Template** (included)

  The template is useful for drawing lines of uniform width and length, especially if graph paper is unavailable. An example is the Helix erasing shield, available at art or drafting stores.

- **Multimeter** or **Ohmmeter*** (digital autoranging preferred)

  The meter must measure electrical resistance to 1–10 megaohms. For example, the VWR-brand Digital Autorange Multimeter P/N 26983-175 is available from VWR at 800/932-5000 for around $30. Multimeters can also be purchased from Radio Shack for $20–40. Another source is parents—many home and car owners have them.

  *If a multimeter or ohmmeter is unavailable, a light emitting diode (LED) and a 9-V battery could be used for several of the activities. See the activity titled "Dependence of Resistance on Dimensions" for details.

- **Silver ink pen**

  Pens that contain conductive silver particles in a liquid medium can be obtained from Mouser Electronics at 800/346-6873, stock number 5168-2200ST (standard tip) or 5168-2200MT (micro-tip); pens are approximately $10 each.

- **Kanthal AF heater wire** (included)

  The Kanthal AF heater wire (75-micron or 3-mil diameter) can be purchased for about $0.03/inch from California Fine Wire at 805/489-5144. Kanthal A1 heater wire is an acceptable alternative.

- **1000-ohm carbon resistor** (included)

  Additional 1000-ohm carbon resistors cost about $0.02 each, and can be obtained from electronics stores, Newark Electronics at 619/453-8211 (part No. 10F3051K), and Digi-Key at 800/344-4539 (part No. 1.0K QBK-ND).

- **Piezoelectric gas ignitor** (included)

  Additional piezoelectric gas ignitors can be obtained for about $0.50 each by contacting Steve Jones at 801/573-8204.

First, You Should Know...

Materials that are intermediate conductors of electricity can be used to demonstrate interesting aspects of the electrical properties of materials. Graphite is one such material. It is called a semimetal because it does not conduct electricity as well as metals like copper, but conducts electricity better than semiconductors (e.g., silicon) or insulators (e.g., most ceramics and plastics). Lines of graphite, drawn with a pencil on a sheet of paper, can be a simple means of exploring the electrical properties of materials and circuits.

Typical No. 2 pencil "lead" consists predominantly of graphite mixed with clay (some wax or polymer may also be present). The clay component is an insulator, so that when a line is drawn, it consists of electrically conductive graphite particles mixed with electrically insulating clay particles. This mixture makes it difficult to perform the activities in this booklet since sometimes the multimeter probe will contact an electrically insulating clay region, and other times it will touch an electrically conductive graphite region. For this reason, the activities in this booklet are best performed with pencils made entirely of graphite (refer to the "Supplies" section for specific pencil recommendations).

More can be learned about the materials involved in these activities by studying their structures at high magnifications. Figures 1 and 2 are photographs taken with a scanning electron microscope of a line drawn with a graphite pencil on a piece of paper. The photograph at 100-X magnification (Fig. 1) has a 200-micrometer scale marker, and the photograph at 500-X magnification (Fig. 2) has a 20-micrometer scale marker. Notice the porous/fibrous nature of the paper; this helps to physically lock in and hold the graphite. Note also that the width and thickness of the line are not very well defined. The graphite line appears to be quite featureless in the photos—even at 10,000-X magnification (not shown), discrete graphite particles are not visible.

These photographs should help to clarify the basis for some of the instructions in this booklet. For example, the instructions call for the graphite lines to be drawn heavily and darkly. This is because the paper fibers that bind the graphite also act as an insulator. If the line in the photographs had not been drawn darkly, some of the paper fibers would have been visible within the line. Also, the photographs show strong variations in the dimensions of the graphite line (caused by the indistinct edge). Because resistance varies with physical dimensions, this can cause some normal variation in the resistance measurements that you and your students make.

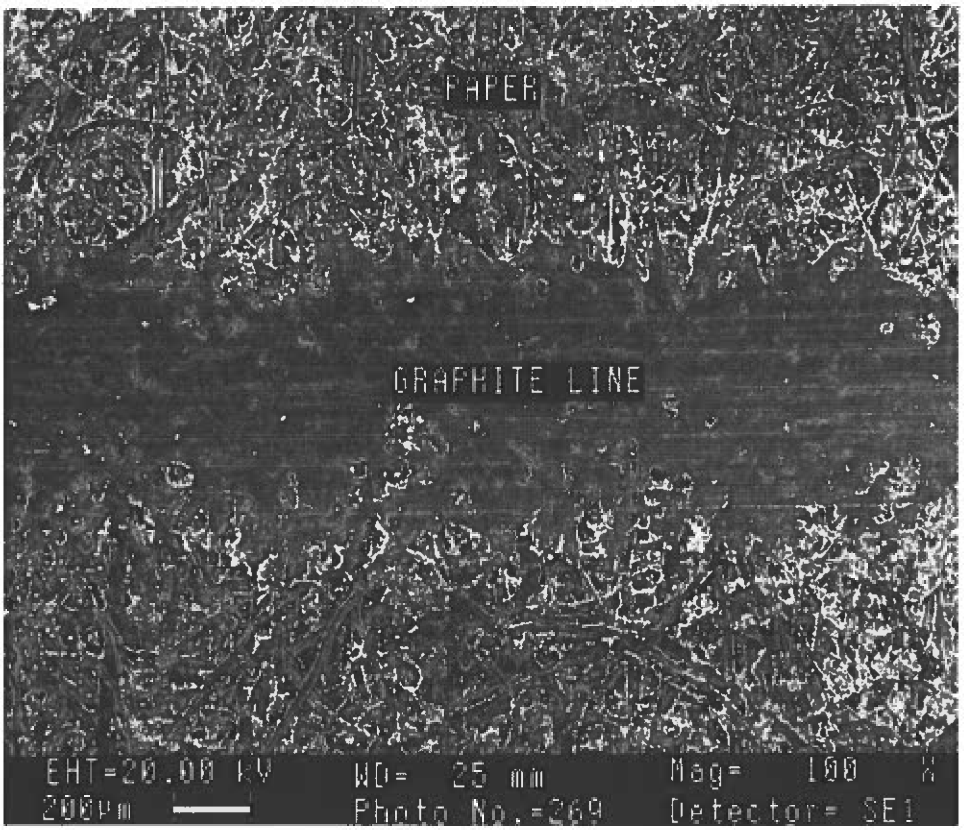

Figure 1. Graphite Line and Paper at 100-X Magnification

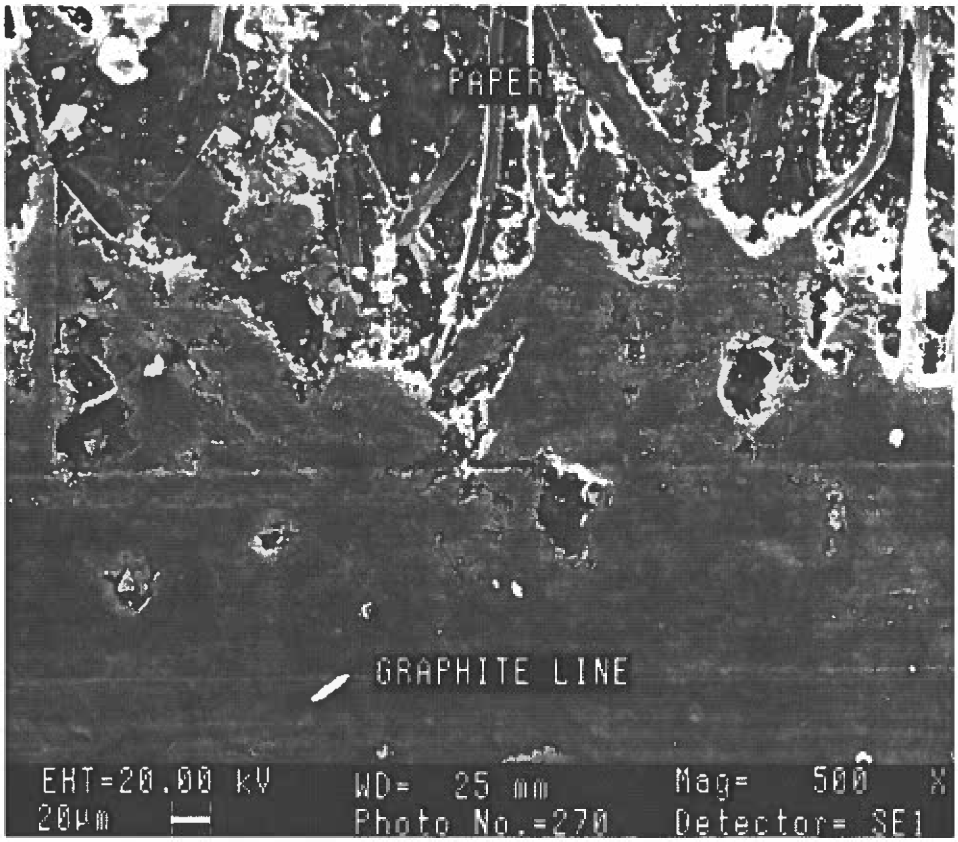

Figure 2. Graphite Line and Paper at 500-X Magnification

Teaching Strategies

In this section, we offer some suggestions for teaching the material presented in this booklet. Through trial and error, we have found techniques that work well with a majority of students. Also, because we know it is hard to be creative every day, we thought some suggestions may be helpful to you.

Before an activity...

- Prepare your students to work in pairs or small groups. Cooperative learning has been proven to enhance student understanding, and at the same time, conserves material resources.
- Try introductory activities that help students integrate the science presented in this booklet with other areas of the curriculum. For example, students could
  - investigate the history and technology of writing implements, such as pencils, pens, erasable pens, crayons, chalk, and white board markers, or
  - investigate how the development of different means for record keeping (i.e., ink, pencils, paper, the printing press, magnetic recording, etc.) affected our society.
- Give the students a pretest to assess their understanding of electrical properties. Try to probe students' existing understanding of electrical properties; students often hold many misconceptions in this area that minds-on activities and teacher assistance can help to correct. The pretest could be in the form of a written exercise, a brainstorming session, a group question/answer session, etc., or you can use the introductory activity to assess students' knowledge.
- The activities require the use of a multimeter or ohmmeter; these devices will probably be a mystery to your students, so give them an opportunity to investigate the meters before jumping into the first activity. (See the Introductory Exercise in the "Student Activities" section. Your students could test and classify a variety of common materials, such as coins, paper, air, plastic, keys, paper clips, rubber bands, etc.) You'll need to clarify how multimeters work before continuing with other activities.*

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*Multimeters are devices that can measure resistance, voltage, and current. The portion of the multimeter that measures resistance is known as an ohmmeter. To measure resistance, the multimeter must have a battery of known voltage connected to an ammeter, which measures the current. The current flow (I) produced from the known voltage (V) is used to calculated the resistance ($R = V/I$) of a closed circuit. The circuit can be closed by touching the probes together (approx. zero resistance) or by touching the probes to the object whose resistance is to be measured (i.e., graphite line or wire, etc.).

- Ask students to draw their conceptions of a circuit. Have them list materials that could be used to make the circuit conduct electricity, as well as materials that could be used to break the circuit. Ask them to explain how the circuit could work, and contemplate how the world (not just the US) would be different without circuits. (Circuits could be mounted on the classroom wall to generate interest in the topic; teachers could use student drawings and explanations to try to make working circuits, then the class could analyze why the circuits did/did not work after doing some of the activities.)

- Decide whether you want to copy and use the student instructions in the back of this book. They can make it easier for your student groups to get started on an activity, while allowing you to monitor learning.

During an activity...

- Be prepared to share some tips with students who are having trouble:
  - graphite lines should be heavy and dark;
  - keep good track of measurement units (for doing calculations) by noting the scale on the multimeter;
  - it is easier to do the graphite line measurements if you start with a line, measure its resistance, then make it longer or wider, and take another measurement; and
  - multimeter probes should be positioned so that the blunt side, and not the sharp point, contacts the graphite lines (the sharp tip can damage the line).

- Get feedback from the students to help gauge what they do and do not understand. This way, you can nudge them in the right direction when they are not on track.

- Provide time for group interaction to discuss predictions, observations, results, and explanations.

- Let each group of students decide how they will keep track of data that they collect. Let them know if you expect them to present the data in the form of tables, charts, or graphs.

- If students have ideas for activity extensions, encourage them to try their ideas out.

After an activity...

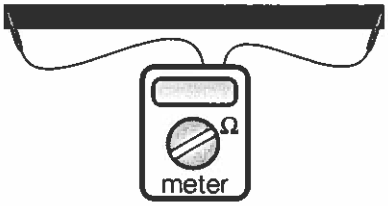
- Try a creative drama to help students visualize what happened on a microscopic level during an activity (see the "Creative Dramas" near the end of the booklet).
- Consider using alternative assessments for evaluation: open-ended discussions, pre- and post-tests, peer reviews, self-tests, performance demonstrations, concept map construction, and portfolios are just a few of the options. For example, students can include their circuit diagrams, measurements, and graphs in portfolios of work to share with their parents.
- Ask students to answer questions (in the form of discussion groups, self-tests, or peer reviews) such as "What have I learned about electrical properties?" "What have I learned about how I learn?"

I. The Basics of Resistance

A. Electrical Resistivity of Graphite

This activity allows you to calculate the electrical resistivity of graphite. You will need a graphite pencil, paper, and a multimeter.

1. Using the graphite pencil, draw a line about 5-cm long and 3-mm wide on a sheet of paper. Keep the width as uniform as possible. Repeatedly draw on the line until it appears completely dark.
2. Now set the multimeter to measure ohms, Ω. Use the 2 multimeter probes to measure the electrical resistance of the line. Touch the blunt edge of a probe to each end of the line.



The greater the value of Ω, the greater the resistance of the line and the less the conductance of the line. The following formula will be useful in understanding the relationship between electrical resistivity and the variables that determine it. The measured electrical resistance (R) of a material is equal to the electrical resistivity (ρ) of the material times the length (L) over which the resistance is measured, divided by the cross-sectional area of the material (A) (See "Mathematical Analogy," right). In other words:

$$R = \rho L/A \qquad (1)$$

Suppose that you measured the resistance of the line to be 3×10^4 ohms, so $R = 3 \times 10^4$ ohms. The distance between the measuring terminals on the line is 5 cm, so $L = 5$ cm. If the line is 3-mm wide then $W = 0.3$ cm. The thickness of the line must be estimated and is on the order of 10 micrometers or 10^{-3} cm, so $T = 10^{-3}$ cm. Therefore, the cross-sectional area $A = WT = (0.3\text{ cm})(10^{-3}\text{ cm}) = 3 \times 10^{-4}\text{ cm}^2$. Now ρ can be calculated:

$$\rho = RA/L = (3 \times 10^4\text{ ohm})(3 \times 10^{-4}\text{ cm}^2)/(5\text{ cm}) = {}^\sim 2\text{ ohm-cm} \qquad (2)$$

The electrical resistivity of graphite can range from about 10^{-4} ohm-cm to 10 ohm-cm. Values of 0.1 – 10 ohm-cm are typically measured in these experiments.

Activity Extensions

1. Compare the electrical resistivities of different types of graphite pencils.
2. Compare resistivities of graphite and standard pencils, such as #1, #2 or #6B, which contain graphite mixed with clay.
3. Compare resistivities of graphite and colored pencils, crayons, and other markers that do not contain graphite.

Mathematical Analogy

The flow of electrons through a wire or resistor of length (L) and cross-sectional area (A) is analogous to the flow of marbles down a tube of length (L) and cross-sectional area (A). One side of the tube is on the ground, and the other side is raised a distance (H) above the ground. A large bag of marbles is situated at the raised tube opening. The flow of marbles will be proportional to the height (H) and inversely proportional to the length (L) of the tube. Therefore,

$$\text{flow of marbles} = \sigma H/L \qquad (3)$$

where the proportionality constant (σ) is the conductivity of the tube. The flow of marbles is equal to the number of marbles passing through the tube per cross-sectional area of the tube per second, so

$$\text{flow of marbles} = \text{no. of marbles}/(A)(\text{second}) \qquad (4)$$

The marble current (I) is defined as the number of marbles that flow per second, so (4) becomes

$$\text{flow of marbles} = I/A \qquad (5)$$

Equating equations (3) and (5) yields

$$I/A = \sigma H/L \qquad (6)$$

Now, the conductivity (σ) is equal to $1/\rho$, where ρ is the resistivity. Equation (6) then becomes

$$\rho = (H/I)\,(A/L) \qquad (7)$$

The resistance of the tube is equal to H/I (the height, H, increases the energy of the marbles the same way that the voltage, V, increases the energy of the electrons; recall that $R = V/I$). Therefore,

$$\rho = RA/L \quad \text{or} \quad R = \rho L/A \qquad (8)$$

Mathematical Definition

The electrical conductivity (σ) is defined as the proportionality constant between the current density (I/A) and the electric field (E), namely

$$I/A = \sigma E \qquad (9)$$

Since $E = V/L$, then

$$\sigma = (I/V)(L/A) \quad \text{or} \quad \rho = 1/\sigma = (V/I)(A/L) \qquad (10)$$

Since $R=V/I$, then

$$\rho = RA/L \quad \text{or} \quad R = \rho L/A \qquad (11)$$

B. Dependence of Resistance on Dimensions

An extrinsic property of a material depends on its dimensions, whereas an intrinsic property does not. For example, the density of a material is an intrinsic property—it can be looked up in a reference book—but the weight or mass of that material is an extrinsic property because it depends on the dimensions of the material.

The electrical resistivity (or its inverse, the electrical conductivity, σ) is an intrinsic property of a material. The electrical resistance is an extrinsic property in that it depends on the dimensions of the material. From equation (1), we can see that the electrical resistance of a material is proportional to its length and inversely proportional to its cross-sectional area. These properties can be observed using a line drawn with the graphite pencil.

1. Lightly draw a line 3-mm wide and 5-cm long. Draw a second line with the same dimensions as the first, but make it heavy and dark. The thickness of these two lines will be quite different. Measure the resistance of each line. The thicker, darker line has a larger cross-sectional area, and should have a lower resistance than the lightly drawn line.

2. Draw a dark line 3-mm wide and 5-cm long. Place the probes from the meter very close together near one end of the line, and read the resistance on the meter. Now move the probes further apart and measure the resistance. Continue recording the resistance while using a ruler (or the graph paper squares) to measure the distance between readings.

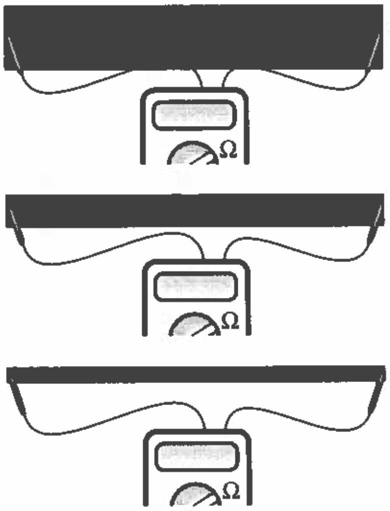
   Try plotting the data as a graph of resistance versus separation distance. The resistance should increase linearly with the separation of the probes because the resistance is proportional to length. Therefore, a straight line graph should result.

   Now determine the slope of the line. Since $R = \rho L/A$, the slope of a graph of R vs. L is ρ/A.

An Inexpensive Alternative to the Multimeter

Instead of using a multimeter, try using a Light-Emitting Diode (LED) and a 9-V battery connected to a snap connector (with red and black wires) to indirectly measure the resistance of the graphite line. Connect the positive (red) wire from the battery to the red wire of the LED by twisting the bare wire ends together. Connect the black wire of the battery to one end of the graphite line, and the black wire of the LED also to the graphite line, next to but not touching the other black wire. Notice how brightly the LED glows when the wires are near each other because the resistance of the graphite between them is small so that a lot of current can flow through the LED. As the wires are separated, but both still touching the graphite line, the LED will glow more dimly, since the resistance of the graphite grows as the separation between the wires increases. This higher resistance causes less current to flow through the LED, so the LED becomes dimmer. This experiment shows that the resistance of a material is proportional to its length.

3. Draw a line about 8-mm wide and 5-cm long. Draw a second line below it about 4-mm wide and 5-cm long. Below the second line, draw a third line about 2-mm wide and 5-cm long. Redraw the lines until they appear black. Try to keep the thickness of the lines comparable. Now place the contacts from the meter at the ends (5 cm apart) of the thin line and measure the resistance. Then place the contacts at the ends of the medium width line and measure the resistance. Finally, place the contacts at the ends of the wide line and measure the resistance.



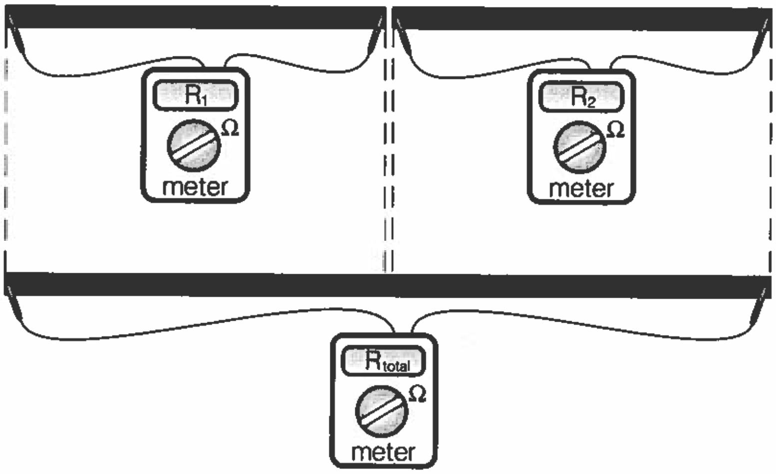
Can you determine the nature of the relationship between resistance and width? (The resistance of the lines should be inversely proportional to their widths.) If desired, make more lines of measured widths, and measure the resistances. Plot the resistance as a function of 1/(width of the lines) to observe the inverse relationship between resistance and cross sectional area. The length of the lines should be at least 5 times their width.

II. Series & Parallel Circuits

You can easily draw series and parallel circuits using the graphite pencil and paper. We will use the multimeter to work with series circuits, first.

A. Resistance of Series Circuits

1. First draw a single resistor, which could be a straight line 3-mm wide and 5-cm long. Measure the resistance of the line. Across from and almost touching the first line, draw another resistor of similar dimensions, and measure its resistance. Now connect the two resistors (fill in the space between them). Measure the resistance across both resistors.



The resistance should equal the sum of the individual resistances. Mathematically, for 2 resistors in series,

$$R_{total} = R_1 + R_2 \tag{12}$$

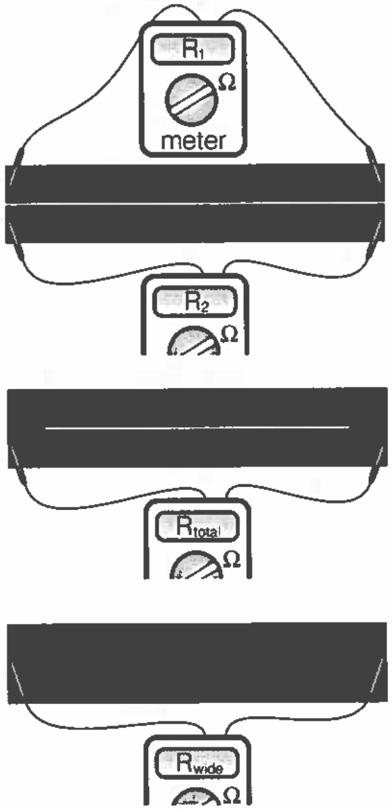
If $R_1 = R_2 = R$, then $R_{total} = 2R$. (For n equal resistors in series, $R_{total} = nR$.) Try drawing a circuit with 3 or more equivalent resistors in series. Measure the resistance of the first resistor (R_1), the first and second resistors ($R_1 + R_2$), and all three resistors ($R_1 + R_2 + R_3$). Now plot these measured resistances versus the number of resistors measured. Note the linear dependence; the slope of the resulting line is R.

The rule for resistors in series is that the total resistance is equal to the sum of the individual resistances. Note that equation (12) is really just a restatement of equation (1): the resistance of a material is proportional to its length. Two equal resistors in series is equivalent to doubling the length of a resistor. Mathematically, for a single resistor:

$$R_1 = \rho L_1/A_1 \tag{1}$$

$$\text{If } L_{total} = 2L_1, \text{ then } R_{total} = 2\rho L_1/A_1 = 2R_1$$

B. Resistance of Parallel Circuits



1. Draw two closely spaced, parallel lines of equal length, say 5-cm long and 5-mm wide. Measure and record the resistance of the first line (R_1), then do the same with the second line (R_2).
2. Now connect the ends of the two lines together and measure the resistance of the resulting parallel circuit (R_{total}).
3. Finally, fill in the remaining white space between the two lines, and measure the resistance of this single, wide line (R_{wide}).

The resistance should obey the standard rule for parallel resistors:

$$1/R_{total} = 1/R_1 + 1/R_2 \tag{13}$$

or

$$R_{total} = (R_1 \times R_2)/(R_1 + R_2)$$

If $R_1 = R_2 = R$, then $R_{total} = R/2$. (For *n* equal resistors in parallel, then $R_{total} = R/n$).

Now try making circuits with 3, 4, 5 or more equivalent parallel resistors, then measure the resistances. Plot R_{total} vs. number of resistors. Notice that the graph is not a straight line. Next, try plotting R_{total} versus 1/(number of resistors). A straight line will result with slope R.

Note that the rule for resistors in parallel is really just a restatement of the relationship that the resistance is inversely proportional to the cross-sectional area of the resistor. The resistance of 2 equal, parallel resistors is the same as a resistor that is twice as wide (twice the cross-sectional area). The double-width resistor shown in step 3 above should have about the same resistance as the parallel combination shown in step 2. Mathematically, for a single resistor

$$R_1 = \rho L_1/A_1 \tag{1}$$

$$\text{If } A_{total} = 2A_1\text{, then } R_{total} = \rho L_1/(2A_1) = R_1/2$$

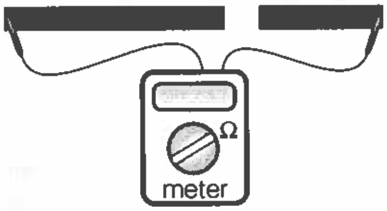
Activity Extensions

1. Assessment Ideas: (a) Draw a line with a resistance of exactly 10,000 ohms. (b) Draw 2 resistors in series that have a resistance of 10,000 ohms. (c) Draw 2 resistors in parallel that have a resistance of 10,000 ohms.
2. Using an insulator, such as a ball-point pen, draw a square on a sheet of paper. Now, using a graphite pencil, try to draw as high a resistance pathway from one corner of the square to the other corner, with the constraints that the line width must be about 3 mm, and the line must be dark. (Some circuit designers must draw long or high resistance lines subject to space constraints.)
3. Draw series and parallel combinations of resistors, starting with individual resistors. Before connecting the individual resistors, measure their resistances. Connect the resistors, predict the resistance of the resultant circuit, then measure the total resistance.
4. Suppose you were a design engineer, and you had to double the resistance of a section of wire on a circuit board. What are the ways that you could accomplish this task?
    a. Double the length of the wire.
    b. Use a wire with half the cross-sectional area.
    c. Use a wire of the identical size, with twice the electrical resistivity of the original wire.
    d. Add an identical wire in series with the first wire.
5. Suppose you were a design engineer, and you had to halve the resistance of a section of wire on a circuit board. What are the ways that you could accomplish this task?
    a. Halve the length of the wire.
    b. Use a wire with twice the cross-sectional area.
    c. Use a wire of the identical size, with half the electrical resistivity of the original wire.
    d. Add an identical wire in parallel with the first wire.

III. Open & Short Circuits

Both open and short circuits can be demonstrated using a graphite line, paper, an eraser, and the multimeter. An open circuit is a conducting path that is broken, while a short circuit is a low-resistance pathway (that could lead to overheating if a voltage source were connected).

1. To make an open circuit, draw a line and measure its resistance. Now erase a section of the line and re-measure its resistance. It should be higher than can be measured by the meter, since the paper is an insulator.

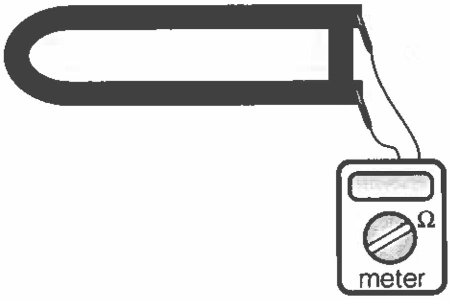


Activity Extension

Make a switch to close your open circuit using graphite-coated paper and a piece of tape or a paper fastener as a hinge. (A switch is a device that opens or closes a circuit.)

2. A short circuit can be demonstrated by drawing an elongated U with the graphite pencil. Measure the resistance of the U.

3. Now draw a line connecting the U near its top. Re-measure the resistance of the U. It is now much lower because the connecting line has short-circuited the majority of the resistance of the U.



IV. More Electrical Resistivity

A. Electrical Resistivity of Silver Ink

This activity allows you to calculate the electrical resistivity of a dried silver-ink line using paper and a multimeter. (See the "Supplies" section to order the pen.)

1. Using the silver-ink pen, draw a line about 5-mm long and 3-mm wide. Allow the ink to dry, then measure the electrical resistance of the line. Calculate the resistivity (ρ), where $\rho = RA/L$.

Lines drawn with this ink are much more conductive than the lines drawn with the graphite pencil. The electrical resistivity of the line will be higher than that of pure silver, which is 1.6×10^{-6} ohm-cm, since the line consists of touching silver particles in an insulating matrix. The resistance of a typical line made with this pen is about 10 ohms, corresponding to an electrical resistivity of about 3×10^{-4} ohm-cm.

Activity Extensions

Try doing some of the other graphite pencil activities, but instead of graphite, use the silver ink pen. Compare the measured resistances.

B. Electrical Resistivity of a Wire

Included with this booklet is a section of Kanthal AF heater wire that is 75 micrometers in diameter. The electrical resistivity of the wire can be calculated using the sample wire and the multimeter.

1. Measure the electrical resistance (R) of a given length of this wire, for example L = 10 cm. Suppose R = 32 ohms. Now you can determine the resistivity (ρ), where $A = \pi r^2$:

$$\rho = RA/L \qquad (2)$$

$$\rho = (32 \text{ ohms})(3.14)(37.5 \times 10^{-4} \text{ cm})^2/10 \text{ cm} = 1.4 \times 10^{-4} \text{ ohm-cm}$$

The electrical resistivity (ρ) of this material is 1.4×10^{-4} ohm-cm. Why did we not include a length of copper wire instead, since copper is the most commonly used electrical conductor? To demonstrate why, calculate the electrical resistance of a 10-cm length of copper wire with a 75-micrometer diameter (for copper, $\rho = 1.7 \times 10^{-6}$ ohm-cm).

$$R = \rho L/A \qquad (1)$$

$$R = (1.7 \times 10^{-6} \text{ ohm-cm})(10 \text{ cm})/[(3.14)(37.5 \times 10^{-4} \text{ cm})^2] = 0.38 \text{ ohm}$$

This is too small a resistance for most multimeters to measure. Most can accurately measure resistances that are at least above a few ohms. Higher resistances can be obtained using smaller diameters (or longer lengths), but 75 micrometers is about the smallest diameter wire that can be easily handled.

V. The Resistance of a Resistor

Electrical resistors based on carbon are commercially available over a wide range of fixed resistance values. The values of the electrical resistance are noted on the resistors using a standardized color code, as shown in the table below. The color-coded value of the resistor that is included with this booklet can be verified using the multimeter.

1. Determine the resistance of the included resistor using the color code printed on the resistor and the table below.

Band Color	**1st Band**	**2nd Band**	**3rd Band**	**4th Band (Tolerance)**
black	0	0	10^0	
brown	1	1	10^1	
red	2	2	10^2	
orange	3	3	10^3	
yellow	4	4	10^4	
green	5	5	10^5	
blue	6	6	10^6	
violet	7	7	10^7	
grey	8	8	10^8	
white	9	9	10^9	
gold	-	-	10^{-1}	± 5%
silver	-	-	10^{-2}	± 10%
no band	-	-	-	± 20%

The resistance (in ohms) of a resistor can be calculated from its colored bands. Begin counting at the band closest to the end of a resistor.

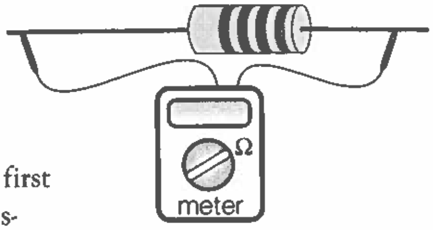
(Start counting on this side.)

If the bands on a resistor are brown, red, green, silver, the resistance is

$$12 \times 10^5 = 1.2 \times 10^6 \text{ ohms (1.2 megaohms)} \pm 10\%.$$

If the bands are red, red, brown, the resistance is $22 \times 10^1 = 220$ ohms.

2. Verify the resistance using the multimeter.
3. Calculate the electrical resistivity of the carbon used to fabricate the resistor. (You must first measure the length and the cross-sectional area of the resistor cylinder.)



VI. Electrical Breakdown

A material that is electrically insulating may become electrically conducting if a sufficiently high voltage is applied across it. The voltage at which the insulating material becomes conductive is called the electrical breakdown voltage, and the corresponding electric field is called the electrical breakdown field.

The electrical breakdown field of air can be exceeded using the piezoelectric gas ignitor included with this booklet. A piezoelectric is a material that produces a voltage across itself when pressure is applied to it. By pushing down on the ignitor, the resultant pressure on the piezoelectric causes a large voltage to be generated across the leads connected to the ends of the piezoelectric material.

The ignitor has two electrodes: a large metallic one and a wire one. To prepare the ignitor for use, strip a small amount of plastic off the end of the wire electrode. Use a thick piece of paper or cardboard to ensure that the spark occurs on the top side of the paper.

> Caution! When using the ignitor, do not touch the bare electrodes and do not put your fingers between the electrodes. Doing so may cause you to be shocked.

A. Electrical Breakdown Field of Air

1. Draw a line 3-cm long using the graphite pencil; leave a 1-mm gap; and continue drawing the line for another 3 cm.
2. Draw a second line in the same way, but with a 3-mm gap. Sequentially draw lines and increase the gap size by 2 mm until a gap of 2 cm is achieved.
3. Starting with the 1-mm gapped line, place the round electrode of the piezoelectric gas ignitor on the line on one side of the gap. Touch the stripped-wire electrode to the line on the other side of the gap. Push down on the ignitor and watch for a spark to jump across the gap. Try the successively larger gaps and see if the spark still jumps across the gap. Determine the largest gap the spark will jump. (It will probably be about 5 mm.)

4. Determine the voltage produced by the ignitor. The electrical breakdown field (E) of air is about 12,000 V/cm. If the length (L) is 5 mm, the voltage (V) produced by the piezoelectric would be:

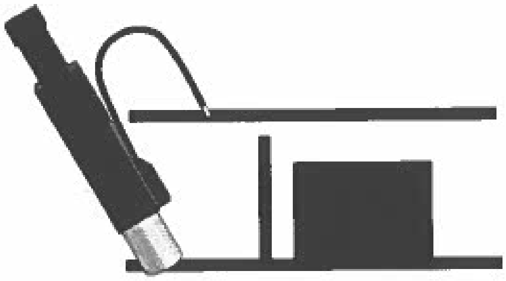
$$V = E \times L = 12{,}000\ \text{V/cm} \times 0.5\ \text{cm} = 6000\ \text{V} \qquad (14)$$

The spark contains sufficient energy to ignite flammable gas; thus, piezoelectric gas ignitors are used in lighters, gas patio grills, and other applications.

B. Path of Least Resistance

The gas ignitor can also be used to demonstrate the principle of a lightning rod.

1. Draw two parallel, horizontal lines 2.5 cm apart. The lower line represents the earth (ground), and the upper line represents the clouds that are the source of the lightning.
2. Draw a graphite rectangular house on the ground (lower line). Next to it, draw a taller, thin line that represents a lightning rod. The lightning rod should touch the ground and be 3 mm from the clouds.
3. Attach one electrode of the ignitor to the clouds, and touch the other electrode to the ground. Push the ignitor. Note that even though the house is much bigger than the lightning rod, the spark (representing the lightning) finds the path of (shortest) least resistance, which is to the lightning rod. This is because the electrical breakdown field across the paper depends only on the distance from one electrode to the other electrode. The distance from the clouds to the lightning rod is less than the distance from the clouds to the house.



Student Activities

Introductory Exercise

1. Take a few minutes to think of what you know about electrical properties. Jot your ideas down.

2. Do you feel you have a good understanding of electrical properties? If you have any uncertainties or questions, write them down, too.

3. Have you used a multimeter or ohmmeter before? Take some time to explore the meter so that you understand how it works (especially on the resistance setting).

4. Try testing the resistance of some everyday materials (i.e., paper, plastic, coins, wood, etc.) using the meter. Before you test each material, predict whether the electrical resistance will be high or low. Explain the reasoning behind your predictions. Were your predictions supported by the experimental data?

5. Try classifying the materials as conductors or insulators based on your tests.

6. Given only a multimeter, a graphite pencil, and a piece of paper, make some predictions about the kinds of activities you can do.

7. Draw some lines on the paper with the graphite pencil. Use the multimeter to measure the resistance of the lines. (Keep track of your results.)

8. Generate a list of questions you can answer by performing tests with the materials you have (e.g., what would the resistance be like if the line were longer?).

9. Can you determine how resistance varies with length of the line? What about the line width? What about the thickness or heaviness of the line?

10. Do your conclusions agree with those of your classmates? If not, take some time to test their ideas, and see if you can come to agreement on how the resistance relates to line dimensions.

11. What other variables, besides line dimensions, might affect the resistance you measure for a particular line? (e.g., wet paper).

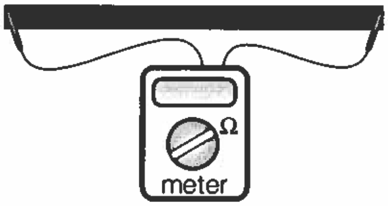
The Basics of Resistance

(Student Activity)

A. Electrical Resistivity of Graphite

This activity helps you to calculate the electrical resistivity of graphite. You will need a graphite pencil, paper, and a multimeter.

1. Using the graphite pencil, draw a line on a sheet of paper. Keep the width as uniform as possible. Repeatedly draw on the line until it appears completely dark. Measure and record the length and width of the line.
2. Use the multimeter to measure the electrical resistance (R) of the line. Record the value, along with its units.



The measured electrical resistance (R) of a material is equal to the electrical resistivity (ρ) of the material times the length (L) over which the resistance is measured divided by the cross-sectional area of the material (A). In other words:

$$R = \rho L/A \qquad (1)$$

3. Solve the formula for electrical resistivity (ρ), then substitute your values for resistance (R) and length (L).
4. Calculate the cross-sectional area (A). It is equal to the width of the line (W) times the thickness of the line (T). Measure the width (in centimeters), and assume the line thickness to be 10^{-3} cm.
5. Now calculate the resistivity, and don't forget your units. Compare your calculation to those of your classmates.

The class values for electrical resistivity should be somewhat similar even though the lines and the values for resistance were different. This is because electrical resistivity is an intrinsic property of a material; thus, it doesn't depend on dimensions.

Activity Extensions

If there is time, try some of these activity extensions.

1. Compare the electrical resistivities of different types of graphite pencils.
2. Compare resistivities of graphite and standard pencils, such as #1, #2 or #6B, which contain graphite mixed with clay.
3. Compare resistivities of graphite and colored pencils, crayons, and other markers that do not contain graphite.

B. Dependence of Resistance on Dimensions

For this activity, you will draw a line, then vary its dimensions to determine how dimensions affect electrical resistance. You will find graph paper (or a ruler) and an eraser useful for this activity.

1. Before you begin to draw, take a moment to predict how varying the (a) length, (b) width, and (c) thickness of a line will affect the electrical resistance.
2. Draw a line and record its dimensions. This will be your "original line." Now measure and record its electrical resistance.
3. Vary the length of the original line, record its new length, and measure its resistance. Do this several times so that you obtain enough data to make a graph that expresses the relationship between line length and electrical resistance.
4. Gently erase everything but the original line. Re-measure the resistance of the line, in case it has changed slightly. Vary the width of the line and measure the width and the resistance. Do this several times so that you obtain enough data to make a graph of width vs. electrical resistance.
5. Finally, erase all but the original line, and vary its thickness—make it darker and lighter. Be sure to measure and record the resistance each time you change the line.
6. Discuss and compare your predictions and the results of your measurements with your classmates.
7. Make graphs of your data for length and width as a way of confirming your conclusions about the relationship between dimensions and resistance. When you graph, don't forget equation (1):

   $$R = \rho L/A, \text{ where } A = WT \qquad (1)$$

   Calculate the slopes of the lines for R versus L and R versus W. Using equation (1), determine the meanings of the slope values.

Series & Parallel Circuits

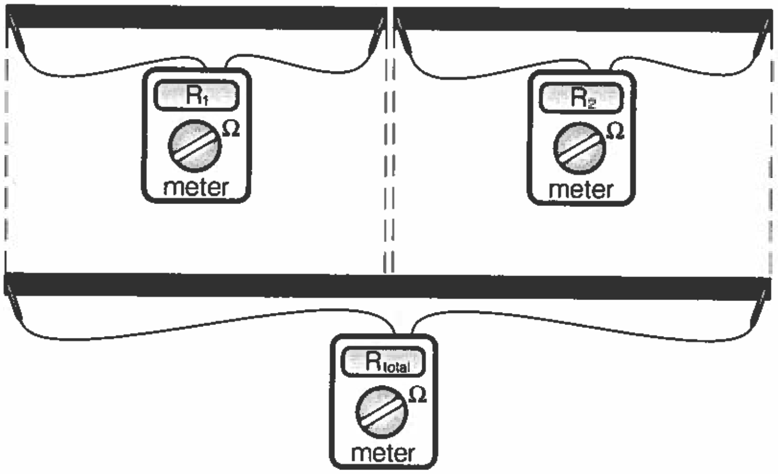
(Student Activity)

This activity will help you compare the electrical resistance of series and parallel circuits.

A. Resistance of Series Circuits

1. Draw 2 lines, as illustrated below. These lines represent 2 resistors in series. Label the resistors R_1 and R_2. Measure and record the resistance of R_1, then do the same with R_2.

2. Connect the 2 resistors (fill in the space between them). Measure the resistance across both resistors (R_{total}) in the "circuit."



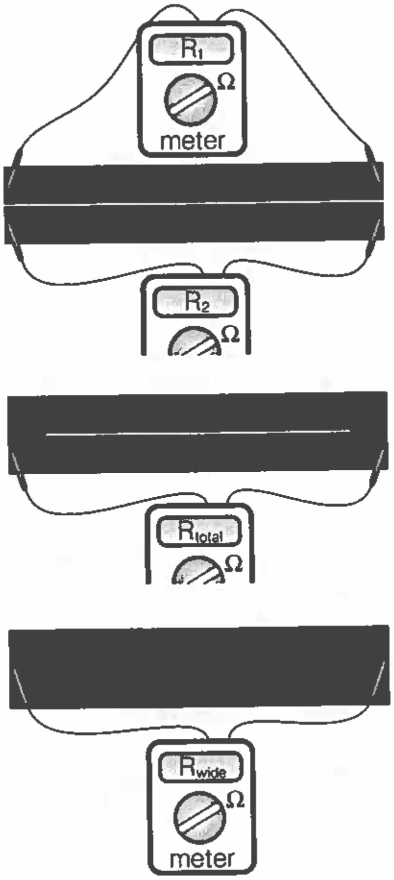
3. What can you determine about the relationship between series resistors and the total resistance of a series circuit? Express this relationship mathematically. (It is customary to use the symbol R for resistance.)

4. What can you determine about the relationship between 2 series resistors and a resistor that is twice as long?

5. Now, try making a circuit with 3, 4, 5 or more equivalent resistors in series. Measure the total resistance of the first resistor ($R_{total} = R_1$), the first 2 resistors ($R_{total} = R_1 + R_2$), the first 3 resistors ($R_{total} = R_1 + R_2 + R_3$), and so on.

6. Graph the values of R_{total} versus the number of resistors measured in each case. Now, plot R_{total} versus 1/(number of resistors). Which graph indicates a linear relationship? Why?

B. Resistance of Parallel Circuits



1. Draw 2 closely spaced, wide, parallel lines (resistors). Measure and record the resistance of the first resistor (R_1), then do the same with the second (R_2).
2. Now connect the edges of the 2 resistors, and measure the resistance of the resulting parallel circuit (R_{total}).
3. What can you determine about the relationship between parallel resistors and the total resistance of a parallel circuit? Express this relationship mathematically.
4. Finally, fill in the remaining white space between the 2 resistors, and measure the resistance of this single, wide resistor (R_{wide}). Compare this value with the total resistance from step 2.
5. What can you determine about the relationship between 2 parallel resistors and a resistor that is twice as wide?
6. Try making a circuit with 3, 4, 5 or more equivalent parallel resistors. Measure the total resistance of the first resistor ($R_{total} = R_1$), the first 2 resistors ($R_{total} = R_1 + R_2$), the first 3 resistors ($R_{total} = R_1 + R_2 + R_3$), and so on.
7. Make a graph of the data by plotting R_{total} versus number of resistors. Now, plot R_{total} versus 1/number of resistors. Which graph indicates a linear relationship and why?

Activity Extensions

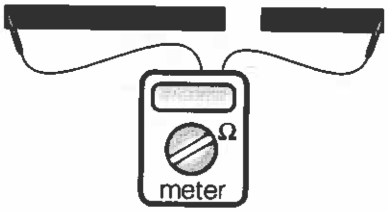
If there is time, try some of these challenging activity extensions.

1. Draw a line with a resistance of exactly 10,000 ohms.
2. Draw 2 resistors in series that have a resistance of 10,000 ohms.
3. Draw 2 resistors in parallel that have a resistance of 10,000 ohms.
4. Draw a square on a sheet of paper. Try to draw as high a resistance pathway from one corner of the square to the other corner, with the constraints that the line width must be about 3 mm, and the line must be dark. (Some circuit designers must draw long or high resistance lines subject to space constraints.)
5. Draw series and parallel combinations of resistors, starting with individual resistors. Before connecting the individual resistors, measure their resistances. Connect the resistors, predict the resistance of the resultant circuit, then measure the total resistance.

Open & Short Circuits

(Student Activity)

Both open and short circuits can be investigated using a graphite line, paper, an eraser, and the multimeter. An open circuit is a conducting path that is broken, while a short circuit is a low-resistance pathway (that could lead to overheating if a voltage source were connected).



1. Draw a line (closed circuit) and measure its resistance. Now erase a section of the line to make an open circuit. Remeasure the resistance.
2. Why does the closed circuit have a different resistance from the open one? Does changing the size of the opening make a difference?

Activity Extension

Design a switch to open and close your circuit. Test your design. If it doesn't work, make modifications until you have a functioning switch.

3. Draw a circuit in the shape of an elongated U. Measure and record the resistance of the circuit.
4. Now, find a way to short the circuit.
5. Why would a short circuit have a lower resistance than before it was shorted?

More Electrical Resistivity

(Student Activity)

A. Electrical Resistivity of Silver Ink

This activity allows you to calculate the electrical resistivity of a dried silver-ink line using paper and a multimeter.

1. Using the silver-ink pen, draw a line about 5-mm long and 3-mm wide. Allow the ink to dry, then measure the electrical resistance of the line.
2. Compare the resistance of the silver ink line to that of a similar graphite line. Is graphite or silver ink a better conductor?
3. Calculate the resistivity (ρ) of the silver ink, where $\rho = RA/L$. (Recall that $A = WT$, where the thickness of the line (T) is 10^{-3} cm. You can use a ruler to determine the values of W and L.)
3. How does the resistivity of silver ink compare to that of graphite and that of pure silver ($\rho = 1.6 \times 10^{-6}$ ohm-cm)? Think of possible reasons for the discrepancy between the resistivities of silver ink and pure silver.

Activity Extensions

Try doing some of the other graphite pencil activities, but instead of graphite, use the silver ink pen. Compare the measured resistances.

B. Electrical Resistivity of a Wire

You can calculate the electrical resistivity of Kanthal AF heater wire that is 75 micrometers in diameter.

1. Measure the electrical resistance (R) of a given length (L) of heater wire.
2. Calculate the resistivity (ρ), where $\rho = RA/L$. (In this case, $A = \pi r^2$.)
3. Compare the resistivity of the Kanthal AF heater wire with that of copper wire ($\rho = 1.7 \times 10^{-6}$ ohm-cm). Why weren't you asked to calculate the resistivity of copper wire instead, since copper is the most commonly used electrical conductor? (Hint: Calculate the resistance for copper wire with the same dimensions as the Kanthal AF heater wire.)

The Resistance of a Resistor

(Student Activity)

Electrical resistors based on carbon are commercially available over a wide range of fixed resistance values. The values of the electrical resistance are noted on the resistors using a standardized color code, as shown in the table below. The color-coded value of the resistor that is included with this booklet can be verified using the multimeter.

1. Determine the resistance of the included resistor using the color code printed on the resistor and the table below.

Band Color	**1st Band**	**2nd Band**	**3rd Band**	**4th Band (Tolerance)**
black	0	0	10^0	
brown	1	1	10^1	
red	2	2	10^2	
orange	3	3	10^3	
yellow	4	4	10^4	
green	5	5	10^5	
blue	6	6	10^6	
violet	7	7	10^7	
grey	8	8	10^8	
white	9	9	10^9	
gold	-	-	10^{-1}	± 5%
silver	-	-	10^{-2}	± 10%
no band	-	-	-	± 20%

The resistance (in ohms) of a resistor can be calculated from its colored bands. Begin counting at the band closest to the end of a resistor.

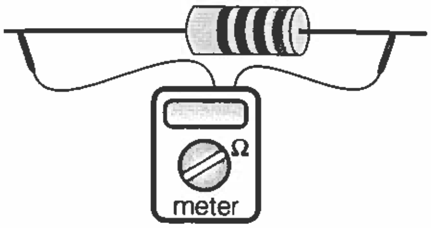
(Start counting on this side.)

If the bands on a resistor are brown, red, green, silver, the resistance is

$$12 \times 10^5 = 1.2 \times 10^6 \text{ ohms (1.2 megaohms)} \pm 10\%.$$

If the bands are red, red, brown, the resistance is $22 \times 10^1 = 220$ ohms.

2. Verify the resistance using the multimeter.
3. Calculate the electrical resistivity of the carbon used to fabricate the resistor ($\rho = RA/L$).



Electrical Break-down

(Student Activity)

A material that is electrically insulating may become electrically conducting if a sufficiently high voltage is applied across it. The voltage at which the insulating material becomes conductive is called the electrical breakdown voltage, and the corresponding electric field is called the electrical breakdown field.

The electrical breakdown field of air can be exceeded using a piezoelectric gas ignitor. A piezoelectric is a material that produces a voltage across itself when pressure is applied to it. By pushing down on the ignitor, the resultant pressure on the piezoelectric causes a large voltage to be generated across the leads connected to the ends of the piezoelectric material. Use a thick piece of paper or a piece of cardboard to ensure that the spark occurs on the top side of the paper.

> Caution! When using the ignitor, do not touch the bare electrodes (a large metallic one and a wire one) and do not put your fingers between the electrodes. Doing so may cause you to be shocked.

A. Electrical Breakdown Field of Air

1. Start a line using the graphite pencil; leave a tiny gap; then continue drawing the line until the gap is in the middle.
2. Place the round electrode of the piezoelectric gas ignitor on the line on one side of the gap. Touch the stripped-wire electrode to the line on the other side of the gap. Push down on the ignitor and watch for a spark to jump across the gap.

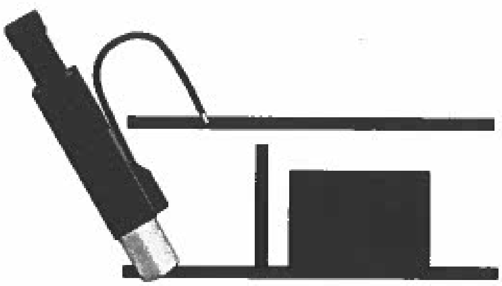
3. Try drawing lines with successively larger gaps to see if the spark still jumps across the gap. What is the largest gap the spark will jump?
4. If the electrical breakdown field (E) of air is about 12,000 V/cm, and (L) is equal to the length (in centimeters) of the largest gap the spark jumped, determine the voltage (V) produced by the piezoelectric (V = E x L).

The spark contains sufficient energy to ignite flammable gas; thus, piezoelectric gas ignitors are used in lighters, gas patio grills, and other applications.

B. Path of Least Resistance

The gas ignitor can also be used to demonstrate the principle of a lightning rod.

1. Take a minute to jot down what you know about lightning rods.
2. Using the graphite pencil, sketch a small, simple illustration that includes the ground, a house on the ground, a lightning rod next to the house, and clouds just above (but not touching) the rod.
3. Put one electrode of the ignitor on the clouds, and touch the other electrode to the ground. Push the ignitor. A spark (representing lightning) should find the path of least resistance to the ground.



4. What path did your spark take? If there was no spark when you pushed the ignitor, try adjusting your drawing. What kinds of adjustments should help?

Creative Dramas

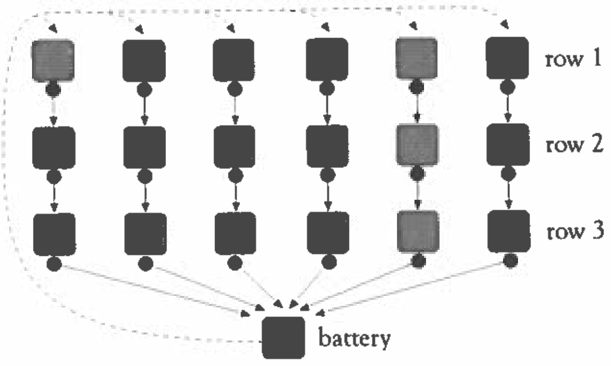
This section provides several creative dramas that can help students better understand the science behind the activities. The dramas provide a way for students to model the flow (or lack of flow) of electrons. In addition, they can be a natural link to the topics of motion, forces, energy conservation, momentum, and potential/kinetic energy. Be sure that the students participating in the dramas, as well as those observing them, understand the processes that the dramas represent.

At the end of each drama is a description of a mechanical analog. Consider collaborating with your school's wood shop to produce the mechanical analogs. Have them add removable dowels to the ramps to simulate scattering centers, which scatter the free electrons, decrease the electrons' drift speed, and lead to an increase in resistivity. A metal with low resistivity, such as copper, would have a low density of scattering centers (dowels). A metal with a high resistivity, such as the Kanthal AF heater wire, would have a high density of scattering centers.

A. Conductor vs. Semiconductor vs. Insulator

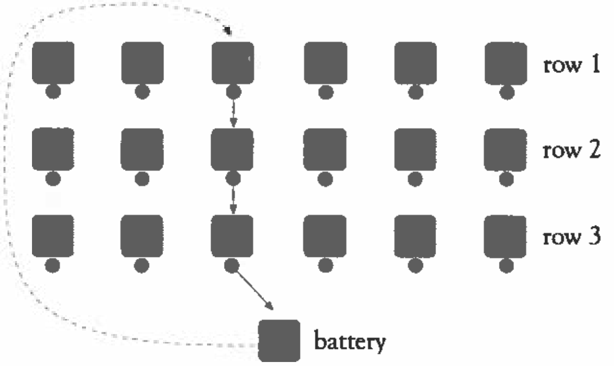
To illustrate conductors, semiconductors or semimetals, and insulators, you will need about 20 balls, such as tennis or ping-pong balls, and 10–19 students. Arrange 18 students so that they stand in 3 rows of 6 abreast. (If you do not have enough students, you may place fewer students abreast.) An additional student should be behind the third row: he/she simulates the battery that supplies energy to the electrons and completes the circuit. Give a ball to each of the students, except the battery student. The movement of the balls will simulate the movement of electrons in each type of material.

1. Conductor. Tell your students that they will begin by simulating an electrical conductor, such as a piece of copper wire. In an electrical conductor, there are many electrons that are free to move. To illustrate the movement of electrons through a conductor, each student should pass his/her ball back to the person behind him/her. (The back row passes their balls to the battery student.)



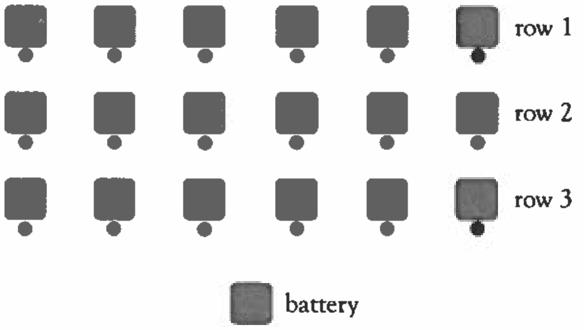
The battery student should then run the 6 balls around to the first row, pass one out to each student, and return to the back. Have the students pass the balls back again. Notice that there is a large flow of electrons—many electrons pass a given point in space in a given time. The battery student collects the 6 balls and passes them out again to the 6 students in the first row.

Time this process for one minute while counting the number of balls that pass a given point in that minute. This number represents the current.



2. Semiconductor or Semimetal. Tell the students that they will now simulate a semiconductor, such as silicon, or a semimetal, such as graphite. In these materials, there are few free electrons and many bound electrons. To illustrate this, only one column of students will help move the free electrons through the material. Tell all but one column of students to hold onto their balls throughout the entire simulation; these balls represent the bound electrons. Have the students in one column pass their balls (free electrons) back to the person behind them.

Have the battery person collect the ball, run it around to the first person in the column of free electrons, and return to the back. Notice that there is a small flow of electrons—few electrons pass a given point in space in a given time. Have the column of students pass their balls back again. Time this process for one minute while counting the number of balls that pass a given point in that minute. This number represents the current.



3. Insulator. Tell the students that they will now simulate an insulator, such as a piece of plastic or rubber. In an electrical insulator, there are essentially no free electrons. Each of the students will hold onto his/her ball and do no passing of the balls. Notice that there is no flow of electrons—no electrons pass a given point in space in a given time. Regardless of the voltage, essentially no current flows in an insulator.

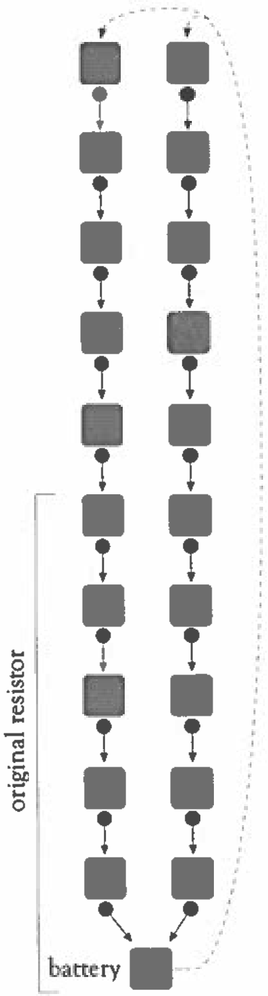
The mechanical analog is to roll many tennis balls in parallel down a sloping piece of wood (conductor), and to roll one tennis ball at a time down a sloping piece of wood (semiconductor or semimetal), and to roll no tennis balls (insulator). The flow of balls is much lower for the semiconductor compared to the conductor. Take the balls and move them back to the top of the piece of wood; repeat the operation. This completes the circuit. The movement of the ball from the bottom to the top of the wood simulates the battery's role of supplying potential energy (voltage) to the electrons.

Note that for the conductor, there is a large flow of balls (electrons) for a given height of the wood (voltage). Since R = V/I, this indicates that for a conductor, R is small since I is large. In contrast, for the semiconductor or semimetal, there is a small flow of balls (electrons) for a given height of the wood (voltage). Since R = V/I, this indicates that for a semiconductor, R is large since I is small. For an insulator, I is essentially zero, so the resistance of an insulator is very large.

B. Length and Width Dependence of Resistance

In this exercise, students will simulate the effects of increasing the length and then the width of a resistor. Assume the same voltage will always be applied across the resistor. You will need 20 balls and 21 students.

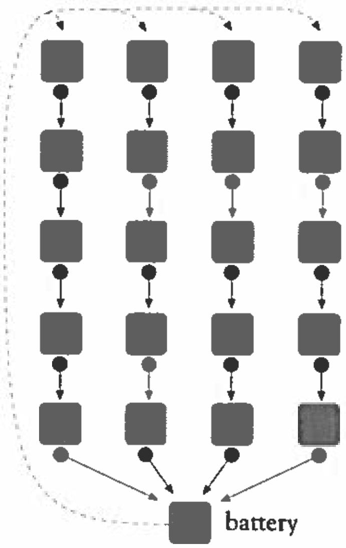
The Original Resistor. To begin, simulate electron flow in a resistor by arranging 10 students into a group that is 2-students wide and 5-students deep. Give a ball to each of the students. An additional student should be at the back of the line: he/she simulates the battery that supplies energy to the electrons and completes the circuit. Have each student pass his/her ball to the student directly behind. The battery student should receive 2 balls from the back row, carry them to the front row, deliver one to each of the 2 students in the first row, and return to the back of the line. Have the students pass the balls back again. The battery person collects 2 balls again, and repeats the process. Observe the flow of balls. In this case, $R_1 = V/I_1$. You can continue the simulation a few times to reinforce the concept of electron flow.



1. Increasing the Length. Increase the length of the original resistor to 10-students long. Give a ball to each of the students. Repeat the situation described above, but now have each of the students hold on to the ball for twice as long before passing it to the student behind him/her. (In other words, have the students pass the balls in slow motion.) Observe the flow of balls. The flow of balls is now half what it was compared to the resistor that was 5-students long. In other words, the current is half what it was, so the resistance of the resistor doubled. In symbols, $R_2 = V/(I_1/2) = 2(V/I_1) = 2R_1$.

Increasing the length of the resistor decreases the electric field across the resistor ($E = V/L$), so there is a smaller force on the electrons ($F = qE$), so they travel at a slower average drift speed, v ($v = \mu E$, where μ is the mobility of the electrons) down the resistor. Thus, the current is reduced ($I = nqvA$, where n is the density of free electrons, and q is the charge on the electron), so the resistance increases ($R = V/I$).

The mechanical analog is best demonstrated by rolling a given number of balls down a sloping piece of wood that contains many dowels (scattering centers). Next roll the balls in the same way down a similar piece of wood that is twice as long, but the ends of the wood are at the same heights (voltage) as before. The balls will now travel at a slower speed–the current of balls is reduced for a given height. In other words, the resistance of the wood increased. Take the balls and move them back to the top of the piece of wood; repeat the operation to complete the circuit. The movement of the balls from the bottom to the top of the wood simulates the battery's role of supplying potential energy (voltage) to the electrons.



2. Increasing the Width. Now increase the width of the original resistor to 4-students wide. Give a ball to each of the students. Repeat the movement of balls as described in the original resistor. The flow of balls is now twice what it was compared to the resistor that was 2-students wide. In other words, the current doubled, so the resistance was halved. In symbols,

$$R_2 = V/(2I_1) = 1/2(V/I_1) = 1/2R_1.$$

Note that in this case, the length of the resistor did not change so the electric field across the resistor also did not change. Thus, the force on the electrons did not change, and they traveled at the same average drift speed down the resistor. The current increased because the number of electrons doubled.

The mechanical analog is to first roll a given number of balls down a sloping piece of wood. Then roll the balls in the same way down a piece of wood that is twice as wide. Keep the concentration (density) of balls on the wood fixed. In other words, you will be rolling twice as many balls down the wider piece of wood compared to the narrower piece of wood. More balls will now travel at the same speed–the current of balls is increased. In other words, the resistance of the wood decreased. Take the balls and move them back to the top of the piece of wood; this completes the circuit. The movement of the balls from the bottom to the top of the wood simulates the battery's role of supplying potential energy (voltage) to the electrons.

C. Summary and Assessment Ideas

Summary. These dramatic illustrations are summarized in the table on the next page. The voltage across the resistor is assumed to remain constant. (Filling in the table below could be used as an assessment tool at the conclusion of the dramas.)

Change in Property	**Effect on Resistance**	**Change in Density of e⁻**	**Change in Speed of e⁻**
Conductor to Insulator	Greatly Increases	Yes; density is essentially zero	Maybe
Conductor to Semiconductor	Increases	Yes; density is much smaller	Maybe
Length Increases	Increases	No	Yes, speed is reduced
Width Increases	Decreases	No; more e⁻ per cross-section*	No

*Another way to view this is to recall the assumption that the voltage across the resistor remained constant. The increase in width increased the flow of charge carriers, so the electrical current increased. Since, according to Ohm's Law, $R = V/I$, if V is constant and I increases, then R decreases.

These illustrations can also be summarized in a slightly different way in the table below.

Change in Property	**Effect on Resistance**	**Change of Current**
Conductor to Insulator	Greatly increases	Greatly decreases
Conductor to Semiconductor	Increases	Decreases
Increase in length	Increases	Decreases
Increase in width	Decreases	Increases

Assessment Ideas. These ideas for assessment/activity extensions involve applying creative dramas to series and parallel circuits. Have your students break into teams of 4 to design

- creative dramas for series and parallel circuits,
- mechanical analogs for series and parallel circuits, and
- summary charts for series and parallel circuits.

Your students can present their designs orally, in written form, and/or as a creative drama exercise with the other members of the class. Note that the assessments for series circuits should be similar to those for increasing the length of a resistor, and the assessments for parallel circuits should be similar to those for increasing the width of a resistor.

Summary of Scientific Principles

The Electrical Conductivity of Materials

Materials have a wide range of electrical properties. Some conduct electricity very well and are called metals. Some, such as graphite, conduct electricity to a lesser extent and may be semimetals or semiconductors. Others conduct almost no electricity and are called insulators. It is important to examine these different materials to better understand differences in electrical properties. For a substance to be a metal, there must be some way for electrons to flow through the material. We can compare metals and insulators to water flowing through a garden hose: if the water (electrons) can flow, we have a metal; if the water (electrons) is frozen, then we have an insulator. In insulators, electrons are "frozen" because they are tightly bound to the individual atoms. In metals, some of the electrons are free to move from atom to atom.

Note: It is important to remember that electron mobility is not the only means by which materials can conduct electricity. In non-metal conductors (e.g., some ionized gases and liquid solutions), conduction can occur due to the migration of electrically charged ions. For the purpose of simplicity, however, this discussion will be limited in scope to metal conductors.

The electrical conductivity of a metal depends on both the density of electrons that are free to flow through the material (generally denoted by the symbol n) and on the ease with which they can flow (mobility, generally denoted by the symbol μ). In our water-hose analogy, these two factors would correspond to the amount of water that flows through the hose and the ease with which it flows. A hose with high mobility would be clean; whereas, a low-mobility hose would be partially clogged.

The electrical conductivity (σ) can be written as the product of the concentration of free electrons (n), their mobility (μ), and their electrical charge (q), so that $\sigma = nq\mu$. In metals, many electrons are free to flow, roughly one per atom, leading to high electrical conductivity. The electrical conductivity is limited by collisions of these electrons with the vibrating atoms in the material. If the electrons collide often, as for the Kanthal AF heater wire, then they will have a low mobility; if they collide less often, as for copper, then they will have a high mobility. This leads to a range of electrical conductivity values for different metals and to conductivity variation with temperature. For insulators, the number of free electrons is orders of magnitude smaller than that of metals, since most of the electrons are bound to the nuclei. This leads to the very low electrical conductivity of insulators, such as most ceramics and plastics.

The Electrical Resistance of Materials

The electrical resistivity (ρ) is equal to the inverse of the electrical conductivity (σ). Metals have high electrical conductivity and low electrical resistivity; insulators have low electrical conductivity and high electrical resistivity. The electrical resistivity is an intrinsic property of a material in that it does not depend on the size or shape of the material.

The electrical resistance, which is the property measured by the ohm-meter/multimeter, is an extrinsic property because it depends on the dimensions of the material. For a bar shaped material, the electrical resistance (R) is equal to the product of the electrical resistivity (ρ) and the length (L) divided by the cross-sectional area (A). Thus, a long, narrow bar would have a higher electrical resistance than would a short, fat bar.

Using our garden hose analogy, the higher the water-pressure difference (voltage) between the ends of the hose, the higher the water flow-rate (current). A long, narrow hose (high resistance) would require a higher water pressure than a short fat hose (low resistance) to yield the same water flow-rate (current).

Energy Bands and Electrical Properties

When individual atoms are placed in a crystal lattice, their discrete electron energy levels widen into electronic energy bands. When the highest occupied energy band is partially filled, the material is a metal. Since the free electrons in this band are responsible for the electrical conductivity of the material, this band is called the conduction band and the electrons are often called conduction electrons. In a metal such as copper, the electrical resistivity at room temperature is low, $\tilde{}$ 2 microohm-cm, and the density of conduction electrons is high, $\tilde{}10^{22}/\text{cm}^3$. The high electrical conductivity of copper is the main reason that it is used for household wiring.

The electrical conductivity of a metal decreases with increasing temperature because of the increased scattering (hence, lower mobility) of the conduction electrons by the more energetically vibrating lattice of atoms (more phonon scattering). In a metal, the density of conduction electrons remains relatively constant with temperature.

When many metallic elements, alloys, and compounds are cooled below a certain temperature, called the superconducting critical temperature (T_c), their conduction electrons undergo a phase transition into the superconducting state. In the superconducting state, a material is a perfect conductor of direct electrical current—it has truly zero electrical resistance. It also displays a unique magnetic behavior called

Graphing Paper (1 square = 5 mm per side)

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About ICE

The Institute for Chemical Education was established in 1983 to provide a center for scientists and science educators to develop and disseminate their ideas for more effective approaches to the teaching of chemistry and science in general. All ICE programs emphasize hands-on science, taught interactively as a means of helping students develop powers of observation and problem solving. ICE aims to stimulate the scientific curiosity of all students, not just those traditionally well served by our educational system.

At Field Center and Affiliate sites across the country, ICE personnel design and conduct workshops that help teachers overcome some of the common obstacles they face in their efforts to deliver first-rate science education. ICE's programs are structured to involve many individuals and a cross-section of the scientific and educational communities. ICE also publishes educational materials that help teachers introduce hands-on, interactive activities in their classrooms and laboratories. Some of ICE's programs and educational publications are outlined below.

Programs

Chem Camps

Middle school students explore the wonders of chemistry through demonstrations and hands-on laboratory experience. There is also a publication with the same title that details how to organize and operate this type of outreach.

SPICE (Students Participating In Chemical Education)

Volunteers experience the joy of presenting chemistry to school groups and the general public.

SSC Workshops

Super Science Connections workshops help K–3 teachers integrate language skills, art, mathematics, social studies, etc. into a hands-on science curriculum.

Educational Materials

Connecting with Chemistry

ICE seeks to maintain the involvement of past workshop participants and others through a semiannual newsletter.

Solid-State Model Kit

The Model Kit can be used to build different crystalline solid structures in a layer-by-layer manner.

Topics in Chemistry

A series of monographs providing teachers with background information on everyday topics in chemistry, such as acid rain and ozone.

Publication 97-002

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